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Professor A. FOWLER, F.R.S., PRESIDENT, in the Chair.

William Wykeham Jacomb, Lieut. R.N., care of H. S. King & Co., 9 Pall Mall, S.W.1;
Rev. Walter A. Mátos, 309 College Avenue, Swarthmore, Pennsylvania, U.S.A.;
Guy B. Petter, 7 Princes Street, Yeovil, Somerset;
Vedaranyesvara Vaidyanatha Ramana Sastrin, Ph.D., F.Z.S., Vedaraniam, Tanjore District., South India;
Herbert William Thomson Roberts, Alpha, Castle Avenue, Broadstairs;
William Edward Sparkes, 8 Claremont Terrace, Sunderland; and
Ernest O. Tancock, Wellington College, Berks,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Charles Henry Adamson, Navigation Master of Pangbourne Nautical College, "Kimpton," Pangbourne (proposed by Wm. R. Cullington);

Amiya Charan Banerji, M.Sc. (Cal.), B.A. (Cantab.), Clare College, Cambridge (proposed by A. S. Eddington);

Henry Jamyn Brooks, The Limes, Shalford Braintree, Essex (proposed by Lord Rayleigh);

The Rev. J. Maurus Moorat, O.S.B., St. Augustine's College, Ramsgate (proposed by A. C. D. Crommelin);

Harry Hemley Plaskett, B.A., Dominion Astrophysical Observatory, Victoria, Canada (proposed by H. F. Newall); and

Arthur Saywell, Head Master of Smith Street (M.) School, Jubilee Street, Mile End Road, E., 94 Kimberley Gardens, Harringay, N. 4 (proposed by T. F. Bowers).

Seventy presents were announced as having been received since the last Ordinary Meeting, including, amongst others:—

Photographs of Jupiter by Mr. J. H. Reynolds (three lantern slides), presented by Mr. Reynolds.

Commencement of the Astronomical Day. Letter from the Admiralty.

(*Communicated by the Secretaries.*)

ADMIRALTY,
19th February 1919.

SIR,—With reference to your letter of 21st December 1919, relative to the substitution in Nautical Publications of a day commencing at midnight for the Astronomical Day, I am commanded by My Lords Commissioners of the Admiralty to inform you that the Superintendent of the *Nautical Almanac* has been directed to make the alterations in the *Nautical Almanac*, commencing at the year 1925.—I am, sir, your obedient Servant,

J. W. S. ANDERSON.

The Secretary,
Royal Astronomical Society:
Burlington House, W.

The Internal Constitution and Radiation of Gaseous Stars.

By J. H. Jeans, M.A., F.R.S.

1. The very striking results which Professor Eddington has been enunciating in recent papers* deserve the fullest discussion. The conclusions he puts forward are substantially the same as those put forward in earlier papers in the *Monthly Notices*.† I have already stated my reasons for dissenting from the reasoning of these earlier papers, but in his last paper Professor Eddington arrives at the same conclusions by a new road. In the present paper, reasons are given to show why the arguments of this paper cannot be accepted, at least without considerable reservations; an attempt is made to examine the question afresh, and conclusions are reached which seem to show that Professor Eddington's main results (but not the reasoning which leads to them) may be accepted provided the star's only source of energy is that of gravitational contraction, but in general not otherwise.

2. The central feature of Professor Eddington's last paper is the assumption‡ that H, the outward flow of energy, is proportional to g, the inward acceleration due to gravity, throughout a star. Of this and other assumptions the only explanation given is that "it will be more easy to justify them after deducing the consequences . . . they appear to be verified very closely by observation."

Professor Eddington does not use the relation $H/g = \text{constant}$ merely as a numerical approximation; he uses it as a fundamental physical equation. His scheme of equations is short by one—an equation which ought to express that the outward flow of energy over any surface is equal to the rate of generation of energy inside, —and the deficiency is made good by the assumed relation that H is proportional to g.

Assume with Professor Eddington that H is proportional to g. Then the outward flow of energy over any sphere of radius r being $4\pi r^2 H$, is proportional to gr^2, and so to the mass inside the sphere. Thus the rate of generation of energy inside any sphere must be proportional to the mass inside the sphere, whence the rate of generation of energy per unit mass must be the same throughout the star, say $4\pi\epsilon$. Without any complicated calculations, it is clear that the rate of emission of energy from a star of mass M must be $4\pi\epsilon M$. The total radiation from a star of given mass will remain constant if ϵ remains constant, but not otherwise.

Professor Eddington tries to show that ϵ will remain constant so long as the ideal gas laws are obeyed. Assuming that H/g remains constant throughout the star, Professor Eddington shows that P, the total pressure, including pressure of radiation, must be of the form

$$P = \kappa\rho^{\frac{4}{3}} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

* "On the Conditions in the Interior of a Star," *Ap. J.*, 48 (1918), 205, and "The Interior of a Star," *Scientia*, 24 (1918 Jan.).

† *M.N.*, 1916 Nov. and 1917 June.

‡ *Ap. J.*, *l.c.*, p. 208.

so that the arrangement of the star will be the same as that of a mass of gas in adiabatic equilibrium with γ, the ratio of the specific heats, equal to $\frac{4}{3}$. Now adiabatic equilibrium with $\gamma=\frac{4}{3}$ presents a very subtle problem. In general, when P and ρ are related by the relation $P=\kappa\rho^{\gamma}$, it is easy to obtain a configuration of equilibrium by starting with any asssumed central density ρ_0 and building up a spherical mass, shell by shell, until the boundary is reached, defined by $\rho=0$. The total mass obtained by summing the masses of all the shells depends on both κ and ρ_0, say

$$M=f(\kappa, \rho_0) \quad . \quad . \quad . \quad . \quad (2)$$

If a given mass M of gas is allowed to form a spherical mass at rest, ρ_0 will of course be determined by this equation. But in the special case of $\gamma=\frac{4}{3}$, $f(\kappa, \rho_0)$ is found to have become independent of ρ_0, and reduces to $f(\kappa)$; there is no equation left to determine ρ_0. The physical interpretation of this mathematical peculiarity is readily seen to be as follows:—If a body of gas for which $P=\kappa\rho^{\frac{4}{3}}$ happens to have a mass M precisely equal to $f(\kappa)$, then it can rest in equilibrium with ρ_0 having any value whatever. If M is less than $f(\kappa)$, the gas will scatter into space ($\rho_0=0$); if M is greater than $f(\kappa)$, the gas will contract until the law $P=\kappa\rho^{\frac{4}{3}}$ is departed from ($\rho_0=\infty$). In a sense, equation (2) still determines ρ_0 when $\gamma=\frac{4}{3}$, but the values of ρ_0 so determined are either zero or infinite, except when M has one very special value.

Professor Eddington's argument is that $\gamma=\frac{4}{3}$, since H/g is assumed constant; therefore $M=f(\kappa)$, so that κ is determined for a star of given mass. From this it easily follows that the emission of radiation is determined for a star of given mass. But H/g is not accurately constant, for Professor Eddington points out (*l.c.*, p. 213) that when $\gamma=\frac{4}{3}$ it varies by 70 per cent. from the boundary to the centre of the star. Thus γ is not accurately equal to $\frac{4}{3}$ in equation (1), and, as soon as γ differs appreciably from $\frac{4}{3}$, the relation $M=f(\kappa)$ disappears altogether—or rather is replaced by equation (2), which merely determines ρ_0, and leaves it quite impossible to obtain any information at all as to the emission of radiation.

For these reasons it seems that Professor Eddington's analysis cannot be said to prove anything definite as to the emission of radiation. This agrees with the obvious common-sense view of the question: something more than the mass must be known about a star before its emission of radiation can be determined.

The Pressure of Radiation.

3. In the interior of a star, the pressure will be the sum of p_G the gas pressure and p_R the pressure of radiation. Thus, with the usual notation, the total pressure P is given by

$$P=\frac{RB}{m}\rho T+\tfrac{1}{3}aT^4 \quad . \quad . \quad . \quad (3)$$

where B is a numerical multiplier introduced to represent deviations from Boyle's law. The gradient of P must satisfy the usual hydrostatic equation

$$\frac{dP}{dr} = -g\rho \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where g, the value of gravity, is given by

$$g = \frac{GM_r}{r^2} = \frac{4\pi G}{r^2}\int_0^r \rho r^2 dr \quad . \quad . \quad . \quad (5)$$

Here G is the gravitation constant, and M_r denotes the mass inside a sphere of radius r.

Integrating equation (4) with respect to r,

$$P = 4\pi G\int_r^a \frac{\rho}{r^2}\int_0^r \rho r^2 dr \quad . \quad . \quad . \quad (6)$$

where $r = a$ is the boundary of the star at which $P = 0$.

Equations (3) and (6) provide only two equations connecting the three quantities P, ρ, and T in an equilibrium configuration. Before these three quantities can be determined, a third equation is needed, and this equation is necessarily very complicated, depending upon the whole past history of the star as well as upon its mechanism of generation of energy. It is to avoid this intricate equation that Professor Eddington introduces his assumed relation $H/g = \text{constant}$. We shall avoid it for the moment by supposing ρ to be given at every point of the star. A comparison of (3) and (6) then enables us to determine T at every point, and hence the pressure of radiation p_R which is equal to $\frac{1}{3}aT^4$.

As an illustrative example, suppose that ρ is given by

$$\rho = \rho_0\left[1 - \left(\frac{r}{a}\right)^n\right] \quad . \quad . \quad . \quad . \quad (7)$$

where ρ_0 is the density at the centre. An enormous range of laws of distribution can be represented by allowing n to vary from ∞ to 0. When $n = \infty$, the density is everywhere equal to ρ_0, so that the mass is homogeneous; when $n = 0$, the whole mass is concentrated at the centre (Roche's model). For any value of n, the mean density $\bar{\rho}$ is given by

$$\bar{\rho} = \frac{n}{n+3}\rho_0 \quad . \quad . \quad . \quad . \quad (8)$$

From equation (5) we find

$$g = 4\pi G\rho_0\left[\frac{r}{3} - \frac{r^{n+1}}{(n+3)a^n}\right],$$

whence

$$P=\frac{2}{3}\pi G\rho_0^2a^2\left[\left(1-\frac{r^2}{a^2}\right)-\frac{2(n+6)}{(n+2)(n+3)}\left\{1-\left(\frac{r}{a}\right)^{n+2}\right\}+\frac{3}{(n+1)(n+3)}\left\{1-\left(\frac{r}{a}\right)^{2n+2}\right\}\right] \quad . \quad (9)$$

The value of P at the centre, say P_0, is

$$P_0=\frac{2}{3}\pi G\rho_0^2a^2\frac{n^2(n+4)}{(n+1)(n+2)(n+3)},$$

or, in terms of the mean density $\bar{\rho}$,

$$P_0=\frac{2}{3}\pi G\bar{\rho}^2a^2\frac{(n+3)(n+4)}{(n+1)(n+2)} \quad . \quad . \quad . \quad (10)$$

Formulæ (9) and (10) depend on the special law of density (7). In general, assume that

$$P=\frac{2}{3}\pi G\bar{\rho}^2a^2\theta \quad . \quad . \quad . \quad (11)$$

Then θ is a numerical multiplier which vanishes at the boundary and has its maximum value θ_0 at the centre. When ρ is given by equation (7), the value of θ_0 is

$$\theta_0=\frac{(n+3)(n+4)}{(n+1)(n+2)}.$$

Throughout the whole range represented by letting n vary from ∞ to 0, θ_0 only varies from 1 to 6. Thus θ_0 does not seem likely to depend greatly upon the law of distribution of density, and may be supposed always to be comparable with unity, its value for a mass of uniform density.

4. Let the gas pressure p_G at any point of the star be put equal to βP, so that β is a number less than unity. Then

$$\beta P=\frac{RB}{m}\rho T\,;\ (1-\beta)P=\tfrac{1}{3}\alpha T^4.$$

Eliminating T and P between these equations and (11),

$$(1-\beta)=\beta^4\left[\frac{\pi}{18}\theta^3\alpha G^3M^2\left(\frac{m}{RB}\right)^4\left(\frac{\bar{\rho}}{\rho}\right)^4\right] \quad . \quad . \quad (12)$$

Let $\odot$ denote the sun's mass (2×10^{33}) and m_h the mass of the hydrogen atom. Introduce the numerical values $\alpha=7{\cdot}06\times10^{-15}$, $G=6{\cdot}66\times10^{-8}$, $m_h/R=1{\cdot}21\times10^{-8}$. Then equation (12) becomes

$$(1-\beta)=\beta^4\left[\frac{1}{32}\theta^3\left(\frac{\bar{\rho}}{B\rho}\right)^4\right]\left(\frac{m}{m_h}\right)^4\left(\frac{M}{\odot}\right)^2 \quad . \quad . \quad (13)$$

This equation, it will be noticed, is perfectly general, and involves no simplifying assumptions or approximations at all.

5. The coefficient of β^4 on the right of equation (13) depends on r, the distance from the centre of the star, through the factor $\theta^3/B^4\rho^4$. If the star is of such low density that Boyle's law is everywhere obeyed, and if the distribution of mass is the same as in a star in adiabatic equilibrium with $\gamma=\frac{4}{3}$, we have $B=1$ and $P\propto\rho^{\frac{5}{3}}$, so that θ^3/ρ^4 is constant. Thus in this special case equation (13) becomes independent of r, reducing to Eddington's equation*

$$(1-\beta)=0{\cdot}00260\beta^4\left(\frac{m}{m_\hbar}\right)^4\left(\frac{M}{\odot}\right)^2 \quad . \quad . \quad (14)$$

We shall not, however, limit ourselves to the supposition that the star is arranged in adiabatic equilibrium with $\gamma=\frac{4}{3}$. We may perhaps regard this arrangement as representing the extreme degree of central condensation which is likely to occur. At the other end of the scale we may examine the case of a mass of uniform density. For this $\bar{\rho}/\rho=1$ at all points; θ is equal to 1 at the centre (*cf.* § 3), and falls off proportionately to P as we pass to the boundary. At the centre equation (13) becomes

$$(1-\beta)=\frac{1}{3^2B^4}\beta^4\left(\frac{m}{m_\hbar}\right)^4\left(\frac{M}{\odot}\right)^2 \quad . \quad . \quad . \quad (15)$$

When $B=1$, the numerical factor becomes $\frac{1}{3^2}$, which is twelve times as great as Eddington's factor ·00260, showing that in a star in which Boyle's law is obeyed, pressure of radiation is likely to be somewhat more important than would appear from Eddington's calculations. When Boyle's law is not obeyed throughout, the factor $\frac{1}{3^2}$ in equation (10) is reduced by the occurrence of a factor B^4 in the denominator, so that even moderate departures from Boyle's law would reduce pressure of radiation back again to the importance assigned to it by Eddington.

It seems fairly certain that m must be quite close to 2 in the interior of any real star, so that the factor $\frac{1}{3^2}(m/m_\hbar)^4$ in equation (15) will not differ greatly from unity. The degree of precision warranted by our calculations will perhaps be expressed by saying that pressure of radiation is comparable with gas pressure in the interior of a star of mass near to that of our sun; it is insignificant in stars of mass much less than that of our sun, and predominates entirely over gas pressure in stars of mass many times (say 100 times) that of our sun. If the star happens to be arranged like a mass in adiabatic equilibrium with $\gamma=\frac{4}{3}$, the ratio of pressure of radiation to total pressure will be the same at all points of the star; but for stars in which the density is more uniform than this, the importance of radiation pressure will diminish as the boundary is approached.

The Emission of Radiation.

6. Professor Eddington claims to have shown† that the emission of radiation from an ideal gaseous star will not vary as it contracts,

* *Ap. J.*, **48** (1918), 210. † *L.c.*, p. 210.

provided that its opacity remains constant, but his proof is dependent on the supposition that H/g is constant. Even if his argument was irreproachable it would only push the problem one stage back; it is just as difficult to decide whether H/g is constant in a typical star as to decide whether the radiation remains constant. I have already shown,* without any such supposition as the constancy of H/g, that the emission would remain constant provided the star's energy came solely from gravitational contraction, but my proof neglected the pressure of radiation in a star's interior, and so was only applicable to a star of small mass.

Consider the general problem in which pressure of radiation is appreciable and may even be predominant. The radiation will depend on the star's past history, and so on the initial conditions of its birth. But we may reasonably assume that after a sufficient lapse of time the influence of these initial conditions becomes obliterated, so that the radiation does not depend on the quantities which specified the star's initial state. We shall also assume for the present that the only source of energy is gravitational contraction.

7. The method of "physical dimensions" is peculiarly suited to the discussion of problems such as this. Let L, M, T, Θ refer to physical dimensions in length, mass, time, and temperature. Then E, the emission of energy per unit time, is of dimensions $\mathsf{M}\mathsf{L}^{-2}\mathsf{T}^{-3}$. In virtue of our suppositions that E does not depend on the initial state of the star, and that the only source of energy is gravitational, it follows that the only quantities which can enter into a formula for E are the following:—†

M, the mass of the star, dimensions	M.
a, the radius ,, ,,	L.
c, the opacity ,, ,,	$\mathsf{M}^{-1}\mathsf{L}^{2}$.
R/m, the gas constant, ,,	$\mathsf{L}^{2}\mathsf{T}^{-2}\Theta^{-1}$.
G, the gravitation constant ,,	$\mathsf{M}^{-1}\mathsf{L}\mathsf{T}^{3-2}$.
σ, the radiation constant (Stefan's), dimensions	$\mathsf{M}\mathsf{T}^{-3}\Theta^{-4}$.
V, the velocity of light, dimensions	$\mathsf{L}\mathsf{T}^{-1}$

Here are seven quantities involving four dimensions. It must be possible to combine them so as to form three independent quantities, each of dimensions zero. These three quantities may be taken to be

$$\frac{cM}{a^2}, \quad \frac{GM}{aV^2}, \quad \frac{\sigma}{V}\left(\frac{m}{R}\right)^4 G^3M^2.$$

Any function of these three quantities is a pure number, and any function of the seven quantities M, a, c, R/m, G, σ, V which is a pure number must necessarily be a function of these three

* *M.N.*, 78 (1917), 36.

† Quantities which are simple combinations of these quantities are, of course, not entered separately. Such are α, which is equal to $4\sigma/V$, and C_p, C_v, the specific heats, which are numerical multiples of R/m.

quantities. The physical dimensions of E are the same as those of GVM/c, whence it follows that E must be of the form

$$E = \frac{GVM}{c} f\left[\frac{cM}{a^2}, \frac{GM}{aV^2}, \frac{\sigma}{V}\left(\frac{m}{R}\right)^4 G^3M^2\right] \quad . \quad . \quad (16)$$

where f denotes a function, as yet undetermined.

Suppose that at any instant in the star's history the values of σ and V are suddenly and simultaneously halved, the values of M, a, c, R/m, and G remaining unaltered. Suppose the star's motion of contraction to be so slow that its velocity may be neglected, every configuration through which the star passes being treated as one of equilibrium. When σ and V are halved, the flow of radiation throughout the star and from its surface are halved, but configurations which were of equilibrium before σ and V were halved remain configurations of equilibrium. Thus the evolution of the star follows precisely the path it would have followed if σ and V had not been halved, but its rate of progress along this path is halved. It follows that when σ and V are halved E must also be halved. Halving σ and V in formula (16), it appears that f must be independent of GM/aV^2. Thus formula (16) must be replaced by

$$E = \frac{GVM}{c} f\left[\frac{cM}{a^2}, \frac{\sigma}{V}\left(\frac{m}{R}\right)^4 G^3M^2\right] \quad . \quad . \quad (17)$$

The opacity c is by definition such that a ray of light passing through a thickness x of matter of density ρ is reduced in the ratio $exp(-c\rho x)$. Thus a ray of light passing from the surface to the centre of the star is reduced in the ratio $exp\left(-c\int_0^a \rho dr\right)$.

The index of the exponential is of the order of cM/a^2, so that this must be a very large quantity for all stars. Thus in evaluating expression (17) we may legitimately pass to the limit in which $cM/a^2 = \infty$. This is equivalent to supposing the radius of the star to be very great compared with the depth we can see into the star.

Introducing this approximation is clearly the same thing as passing to the limit $c = \infty$. In this case E will plainly vary inversely as c, so that equation (17) assumes the form

$$E = \frac{GVM}{c} f(x) \quad . \quad . \quad . \quad . \quad (18)$$

where

$$x = \frac{\sigma}{V}\left(\frac{m}{R}\right)^4 G^3M^2 \quad . \quad . \quad . \quad . \quad (19)$$

Formula (18) shows that E is independent of a; in other words, E does not change as the evolution of the star progresses, so long as the laws of an ideal gas are obeyed.

8. In a general investigation from physical dimensions such as the present, it is impossible to determine the form of the function $f(x)$, so that E cannot be completely determined. Comparison with

equation (12) shows that x is proportional to $(1-\beta)/\beta^4$, and so depends on the ratio of radiation pressure to total pressure at the centre of the star. For a star in which radiation pressure is negligible, β will be equal to unity, so that $x=0$. Thus the emission of such a star may be determined by passing to the limit and taking $x=0$. This is equivalent to passing to the limit in which σ is infinitesimal; it is clear that E will be proportional to σ, so that $f(x)$ reduces to Cx, where C is a constant, and the value of E becomes

$$E=\frac{GVM}{c}Cx=C\frac{\sigma}{c}\left(\frac{m}{R}\right)^4G^4M^3 \quad . \quad . \quad (20)$$

We see that for stars of mass so small that the pressure of radiation is negligible in comparison with gas pressure, E varies as M^3. A magnitude difference in brightness corresponds to a ratio 1·36 in mass.

9. Consider next a star which has other sources of energy than its gravitational contraction. First suppose that the star generates energy at a uniform rate $4\pi\epsilon$ per unit mass, independently of the pressure, density, etc. The foregoing argument from physical dimensions still holds, subject to the appropriate modifications. In addition to depending on the seven quantities enumerated in § 6, E can also depend on $4\pi\epsilon$, the rate of energy-generation, dimensions L^2T^{-3}.

On making the necessary modifications in the argument, it is found that formula (18) must be replaced by

$$E=\frac{GVM}{c}f\left[x,\frac{4\pi\epsilon c}{GV}\right] \quad . \quad . \quad . \quad (21)$$

Professor Eddington claims, or at least his proof seems to claim, to show that if ϵ were suddenly halved, E would be unaltered. This could only be the case if E were independent of ϵ. Now E cannot be independent of ϵ, for when ϵ is infinitely great the limiting value of E must necessarily be $4\pi\epsilon M$. Indeed, it seems obvious without discussion that if ϵ varies with the time the light-emission of the star must to some extent follow the fluctuations in ϵ; a sudden increase in ϵ must result in increased brightness, and vice versa.

Thus our conclusion is that the radiation from a gaseous star obeying Boyle's law will remain constant provided $\epsilon=0$, so that the whole energy comes from contraction, provided ϵ remains constant, but not otherwise.

10. Some insight into the mechanism of emission can be obtained as follows:—Consider a star contracting homologously from radius a to radius a'. Let the pressure, density, and temperature at any point be changed by this contraction from P, ρ, T to P', ρ', T'. Since the contraction is homologous,

$$\rho a^3=\rho' a'^3. \quad . \quad . \quad . \quad . \quad (22)$$

The mechanical equations of equilibrium must be satisfied both before and after contraction. These may be put in the form

$$\frac{\partial P}{\partial r} = -\gamma\rho = -\frac{M_r}{r^2}\rho,$$

whence, by comparison with (22),

$$Pa^4 = P'a'^4 \quad . \quad . \quad . \quad . \quad (23)$$

Inserting the value for P from equation (3), and taking $B = 1$ to represent the case of a perfect gas, this gives

$$\frac{R}{m}(\rho a^3)Ta + \tfrac{1}{3}a(Ta)^4 = \frac{R}{m}(\rho' a'^3)T'a' + \tfrac{1}{3}a(T'a')^4,$$

so that

$$Ta = T'a' \quad . \quad . \quad . \quad . \quad (24)$$

This is Lane's law. Thus it appears that Lane's law remains valid when pressure of radiation is taken into account.

Finally, the equation of radiative transfer of energy must be satisfied. Assuming the energy to come solely from contraction, this is readily found to be *

$$r^2(\rho C_v + \tfrac{4}{3}aT^3)\frac{\partial T}{\partial t} = \frac{\partial}{\partial r}\left(\kappa r^2 \frac{\partial T}{\partial r}\right) \quad . \quad . \quad (25)$$

in which κ is the coefficient of radiative conduction. Still assuming that the only source of energy is gravitational, the value of κ, except at points quite close to the boundary, is †

$$\kappa = \frac{16\sigma T^3}{3c\rho} \quad . \quad . \quad . \quad . \quad (26)$$

In homologous contraction T^3/ρ remains unaltered, so that κ remains unaltered provided that c remains unaltered by contraction. It is not necessary that c should be constant throughout the gas, but only that it should not vary with the time. If this condition is satisfied, it is readily seen that continual homologous contraction is possible provided that $\partial T/\partial t$ remains constant at each point of the gas. This is possible by equation (24) provided $a^{-2}da/dt$ is constant. Thus there is a possible motion in which the star contracts homologously, the reciprocal of its radius diminishing uniformly with the time. It may reasonably be assumed that this would be the actual motion in a star which was so old that the influence of the initial conditions of its birth had disappeared.

The flow of energy per unit area H is equal to $-\kappa\partial T/\partial r$. In the motion just considered $\partial T/\partial r$ varies, by equation (24), as $1/a^2$ for each element of the gas. The acceleration due to gravity, g, also varies as $1/a^2$ for each element of the gas, so that for

* *M.N.*, 78 (1917), 40. † *Ibid.*, 31.

each element H/g remains constant throughout contraction. Eddington assumed H/g to be constant in space; we have found that H/g is not constant in space, but that H/g remains constant in time, provided the only source of energy is gravitational.

Consider an element of the star just so far inside the star that the disturbance due to the proximity of the boundary may be ignored. Let η be the constant value of the ratio H/g at this point. Then

$$H = g\eta = \frac{GM}{a^2}\eta,$$

so that the total emission of energy is

$$4\pi a^2 H = GM\eta, \text{ a constant} \quad . \quad . \quad . \quad (27)$$

Thus the emission of energy remains constant so long as the star obeys the laws of a perfect gas.

This can be seen in another way. The total energy W of a star is the algebraic sum of three quantities—

(i) W_1, the gravitational potential energy (always negative).
(ii) W_2, the heat energy of the molecules of the gas (always positive).
(iii) W_3, the energy of the imprisoned radiation (always positive).

It is readily seen that, as the star contracts homologously, each of these quantities varies as $1/a$, so that

$$W = -\frac{k}{a} \quad . \quad . \quad . \quad . \quad (28)$$

where k depends on the mass, etc., of the star, but does not change as homologous contraction proceeds. The emission is now given by

$$E = -\frac{\partial W}{\partial t} = -\frac{k}{a^2}\frac{da}{dt} \quad . \quad . \quad . \quad . \quad (29)$$

which has already been seen to remain constant.

Bodies of very great Mass.

11. Conjecturally we may regard the nucleus of a spiral nebula as consisting of gas, dust, or other finely divided matter of mass many thousands or millions of times that of the sun. The nuclei are not spherical, but the formulæ we have obtained for spherical masses will give results accurate as regards order of magnitude.

In formula (13), $M/\odot$ may be regarded as a very great quantity, so that β must be very small except for points so near to the boundary that θ is very small. In other words, the pressure of radiation must preponderate over all other pressures in the interior of a very massive body. This is true whether Boyle's law is accurately obeyed or not.

In such a mass (except near the boundary) we may take $\beta = 0$ as an approximation. Thus we have

$$P = \tfrac{1}{3}aT^4, \quad \frac{\partial P}{\partial r} = -g\rho,$$

so that

$$H = -\frac{16\sigma T^3}{3c\rho}\frac{\partial T}{\partial r} = -\frac{4\sigma}{ac\rho}\frac{\partial P}{\partial r} = -\frac{V}{c\rho}\frac{\partial P}{\partial r} = \frac{Vg}{c} \quad . \quad (30)$$

A body of very great mass is probably broken up almost entirely into its constituent electrons except quite near to the surface. Thus c is likely to be constant throughout the interior of a star, except in regions close to the surface. Incidentally, this shows also that the effective value of m is likely to be constant throughout the interior of a star, m/m_0 being effectively equal to 2, as already assumed.

Assuming c to be constant throughout the interior of the mass, equation (30) shows that H/g is constant.* The total emission of energy is

$$E = 4\pi a^2 H = 4\pi a^2 V g/c = 4\pi G V M/c \quad . \quad . \quad (31)$$

This is obviously the limit of formula (18) with $x = \infty$.

12. The appropriate modifications for a non-spherical body are easily found. Let Ω denote the total gravitational potential at any point, including that of tidal forces, if any are present. Let the mass be supposed in rotation with an angular velocity ω about the axis of z, ω being possibly different at different points of the mass. The hydrostatic equations are

$$\frac{1}{\rho}\frac{\partial P}{\partial x} = \frac{\partial \Omega}{\partial x} + \omega^2 x, \text{ etc.} \quad . \quad . \quad . \quad . \quad (32)$$

Still assuming the pressure of radiation to preponderate over all other pressures, we may take $P = \frac{1}{3}aT^4$. The flux of radiant energy per unit area has components parallel to the three axes which are given by

$$\begin{aligned}(H_x, H_y, H_z) &= -\frac{16\sigma T^3}{3c\rho}\left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z}\right) \\ &= -\frac{V}{c\rho}\left(\frac{\partial P}{\partial x}, \frac{\partial P}{\partial y}, \frac{\partial P}{\partial z}\right) \\ &= -\frac{V}{c}\left(\frac{\partial \Omega}{\partial x} + \omega^2 x, \frac{\partial \Omega}{\partial y} + \omega^2 y, \frac{\partial \Omega}{\partial z}\right) \quad . \quad (33)\end{aligned}$$

* This is Eddington's assumption. We have found it to be valid only when the mass is very great. In this special case it is not possible to pass to Eddington's deduction that the mass is in adiabatic equilibrium with $\gamma = \frac{4}{3}$, for H/g is only constant when $\beta = 0$, and in this special case Eddington's proof fails. Indeed, when H/g is constant, the mass cannot be in adiabatic equilibrium with $\gamma = \frac{4}{3}$, for, as Eddington points out, this would cause H/g to vary by about 70 per cent. through the mass.

Thus the flow of energy at any point is proportional to, and in the opposite direction to, the acceleration due to gravity at the point, gravity being the resultant of the attraction of the mass, tidal forces (if any), and centrifugal force.

The total emission of radiation from the surface of the mass is

$$\begin{aligned} \mathrm{E} &= \iint\Big(l\mathrm{H}_x + m\mathrm{H}_y + n\mathrm{H}_z\Big)d\mathrm{S} \\ &= \iiint\Big(\frac{\partial \mathrm{H}_x}{\partial x} + \frac{\partial \mathrm{H}_y}{\partial y} + \frac{\partial \mathrm{H}_z}{\partial z}\Big)dxdydz \\ &= -\frac{\mathrm{V}}{c}\iiint\Big(\nabla^2\Omega + 2\omega^2 + x\frac{\partial\omega^2}{\partial x} + y\frac{\partial\omega^2}{\partial y}\Big)dxdydz \end{aligned}$$

For uniform rotation, this reduces to

$$\mathrm{E} = \frac{4\pi\mathrm{GVM}}{c}\Big(1 - \frac{\omega^2}{2\pi\mathrm{G}\bar{\rho}}\Big) \quad . \quad . \quad . \quad (34)$$

Comparing with formula (31), we see that tidal forces do not alter the total emission of radiation, but alter its distribution. Rotation decreases the total emission in the ratio $1 - \omega^2/2\pi\mathrm{G}\bar{\rho}$. As rotation increases the surface of a body, it follows that a rotating body will appear less luminous than the same body at rest. The brightness will vary from point to point of the surface, being proportional always to the value of gravity (including centrifugal force) at the point.*

Rapidity of Evolution.

13. For a body of very great mass, we have found that the rate of emission (31) is proportional to M; for a body of very small mass, the rate of emission was found to be proportional to M^3. For intermediate masses, we have not been able to determine the exact way in which the rate of emission depends on the mass.

For a star whose mass is very great compared with that of the sun, the total energy ($\mathrm{W}_1 + \mathrm{W}_3$ in § 10) is readily seen to be proportional to M^2/a. The same is true of the total energy ($\mathrm{W}_1 + \mathrm{W}_2$) of a star whose mass is very small compared with that of the sun. Thus k in formulæ (28) and (29) may be replaced by $\mu\mathrm{M}^2$ where μ is independent of the mass for both very great and very small stars, although not for stars of intermediate mass.

Equation (29) now becomes

$$\mathrm{E} = \mu\mathrm{M}^2\frac{d}{dt}\Big(\frac{1}{a}\Big),$$

so that

$$\frac{1}{a} = \frac{\mathrm{E}t}{\mu\mathrm{M}^2}$$

* A real difficulty may emerge from the circumstance that formula (34) would require a nebula of mass a million times that of Canopus to be of luminosity a million times that of Canopus.

where t is measured from the time at which the mass was of zero density ($a = \infty$). Since $\frac{4}{3}\pi\bar{\rho}a^3 = \mathrm{M}$, we find

$$\bar{\rho} = \frac{3}{4\pi}\frac{\mathrm{E}^3}{\mu^3\mathrm{M}^5}t^3.$$

Thus for stars of very great mass, for which $\mathrm{E} \propto \mathrm{M}$, $\bar{\rho}$ will be proportional to t^3/M^2; for stars of very small mass, for which $\mathrm{E} \propto \mathrm{M}^3$, $\bar{\rho}$ will be proportional to $t^3\mathrm{M}^4$. It follows that, among the stars of very great mass, the most massive stars will develop most slowly. Among the stars of very small mass the reverse will be the case, and the stars of smallest mass will develop most slowly. In a miscellaneous collection of stars, those whose mass is either extremely small or extremely great compared with that of the sun will develop very slowly; the stars which live their lives most rapidly will be those of intermediate mass. In a collection of binary systems which have originated by fission, the more massive component may be expected to be the further developed if the system is of small mass (*e.g.* Krüger 60), but the less massive component ought to be the further developed if the system is of great mass (*e.g.* Sirius, ζ Herculis, etc.).

Summary and Conclusion.

14. We have tried to discuss the problem of the internal constitution and radiation of gaseous stars without making Eddington's somewhat enigmatical assumption that H, the flow of radiant energy at any point, is proportional to g, the value of gravity at the point. Making the alternative assumption that energy is produced only by gravitational contraction, we have found that the radiation from a gaseous star will remain constant so long as Boyle's law is obeyed throughout. We have found that the same is true when the star has sources of energy other than gravitational, provided the rate of generation of energy per unit mass does not vary with the time. In all other cases we have found (in opposition to Eddington) that the rate of emission of energy will vary as the development of the star proceeds.

From the circumstance that the absolute magnitudes of giant stars are found to be independent (very approximately) of the stage of evolution of the star, we can obtain some information as to the star's source of energy. Interpreting the evidence as showing that the radiation from a giant star does not vary with the time, it follows that there can be only two kinds of sources of energy—

(i) gravitational energy of contraction,
(ii) sources which emit energy at a uniform rate which is independent of the time.

It is difficult to imagine any source of the second kind. The only source of energy of sufficient power with which I am acquainted is that provided by the annihilation of pairs of positive and negative

electrons: energy is provided by the destruction of matter.* It is almost impossible to believe that the rate of generation of this energy should be entirely uninfluenced by the pressure, temperature, etc., of the star, so that we are almost driven to believing the only source of stellar energy to be gravitational contraction.

15. We may divide gaseous stellar bodies into three classes—

(A) Bodies of mass very great compared with the sun (*e.g.* the nuclei of spiral nebulæ).

(B) Bodies of mass comparable with the sun (*e.g.* most gaseous stars).

(C) Bodies of mass very small compared with the sun (*e.g.* planets, satellites, etc., in the gaseous state).

We have found that in class A pressure of radiation predominates entirely over gas pressure. In such bodies the matter merely provides a net whose function is to imprison radiant energy. In bodies of class B, pressure of radiation is comparable with gas pressure. In bodies of class C, radiation pressure is insignificant. These results agree on the whole with those of Eddington. Our analysis has shown that radiation pressure may be somewhat, but not enormously, greater than was estimated by Eddington.

In bodies of class A, the emission E is proportional to the mass M. As the body contracts, the rate of increase of $\bar{\rho}$, the mean density, is proportional to $M^{-\frac{2}{3}}$.

In bodies of class C, E is proportional to M^3, and $d\bar{\rho}/dt$ is proportional to $M^{\frac{1}{3}}$.

It has not been found possible to obtain exact relations for bodies of class B.

We have found that in a random assortment of stars those of moderate mass would develop most rapidly, stars of very great and very small mass lagging behind. In massive binaries, the less massive component ought to be further advanced in development; the reverse should be the case in binaries of small mass.

Finally, we have investigated the distribution of brightness over the surface of a body of class A in rotation. The total emission from a rotating body is less than that from the same body at rest, and the surface brightness at any point is proportional to the distance a free particle would fall in one second at that point under the influence of gravity, centrifugal force, tidal forces, etc.

* One per cent of the sun's mass, transformed into energy, would provide radiation at the sun's present rate for 150,000 million years.

On the Distribution of the Stars of the 11th Magnitude.

By Dr. A. Pannekoek.

(*Communicated by Professor W. de Sitter, Assoc. R.A.S.*)

1. In the October number of the *Monthly Notices* (78, 668) Mr. Plummer published some results on the distribution of the stars, based on Dr. Nort's reduction of the Harvard Map of the Sky. These results need some corrections, as the data used are not homogeneous.

The Harvard charts have been counted by Mr. Henie at Lund, who also determined the two principal elements for reducing the counted numbers to homogeneous densities, viz. the limiting magnitude for each chart and the decrease of star density with increasing distance from the centre. But, as Dr. Nort in his new reduction of these star counts pointed out, both determinations are unsatisfactory. To find the limiting magnitude—or the reduction of the star densities to the same limiting magnitude,—Mr. Henie made use of the series of comparison stars for variables from Hagen's Atlas; but he used only a part of them, and for most charts the limiting magnitude rests on 3, 2, or 1 series of stars only. The reduction to centre was deduced from the decrease of star density from the centre to the margins. As the Harvard charts extend over 30°, the decrease must be found too great for the charts with great density, having their centre in the Milky Way. A better and indeed sufficient determination of the reduction to centre could be obtained by counting a second set of plates with the centres at the edges of the charts used. With the materials at hand it is, however, not possible to find much better values now; so we must adopt this reduction as computed by Dr. Nort.

For a study of the distribution of the stars over the sky as a whole, the other element, the limiting magnitude for every chart, is much more important. That in this respect the results of Dr. Nort are far from homogeneous may be seen at once when we plot them on a celestial map. The reduced star densities of adjacents charts show, in their overlapping parts, considerable systematic differences. Before using them for statistical studies it is therefore necessary to make them homogeneous.

2. As a first, and for our purpose sufficient, approximation we suppose that the star densities of Dr. Nort can be reduced to a homogeneous system by multiplying them by factors C that are constant for each Harvard chart. These factors may be found from the overlapping parts of the adjacent charts. If d_1 and d_2 are the mean densities on charts a and b in the region they have in common (usually containing 5–15 counted fields), then

$$\log d_1 - \log d_2 = \log C_b - \log C_a.$$

Each chart is in this way connected with all the surrounding charts (at least 4), and from the whole of these equations (there

are 118 of them) we may find log C for the 55 charts by a process of successive approximations that makes their sum zero. As the mean error of a result for log C is 0·05, and as 35 out of 55 values are 0·10 or more and 10 values are 0·20 or more, the reality of the majority of these corrections stands beyond doubt. The coefficients thus found are contained in Table I.

TABLE I.

Coefficients by which the Star Densities of Dr. Nort must be multiplied.

Chart.	Centre.		Coefficient.	Chart.	Centre.		Coefficient.
1	... °	+90°	1·05	29	210°	0°	1·38
2	0	+60	0·76	30	240	,,	0·69
3	45	,,	2·19	31	270	,,	0·60
4	90	,,	1·41	32	300	,,	1·23
5	135	,,	1·02	33	330	,,	0·81
6	180	,,	1·35	34	0	−30	0·74
7	225	,,	1·26	35	30	,,	0·65
8	270	,,	1·35	36	60	,,	0·93
9	315	,,	1·38	37	90	,,	0·78
10	0	+30	0·83	38	120	,,	1·26
11	30	,,	0·81	39	150	,,	1·38
12	60	,,	0·98	40	180	,,	1·02
13	90	,,	0·69	41	210	,,	0·85
14	120	,,	0·48	42	240	,,	0·69
15	150	,,	0·63	43	270	,,	0·81
16	180	,,	1·29	44	300	,,	0·56
17	210	,,	1·10	45	330	,,	0·93
18	240	,,	1·20	46	0	−60	0·55
19	270	,,	1·02	47	45	,,	0·65
20	300	,,	1·29	48	90	,,	0·68
21	330	,,	0·74	49	135	,,	0·76
22	0	0	0·66	50	180	,,	0·66
23	30	,,	1·07	51	225	,,	0·63
24	60	,,	1·05	52	270	,,	1·17
25	90	,,	0·79	53	315	,,	0·66
26	120	,,	1·17	54	0	−85	0·55
27	150	,,	1·70	55	210	−75	0·52
28	180	,,	1·51				

The changes that are brought about by these coefficients in the star densities are rather considerable. This means, that without these corrections the numbers of Dr. Nort must give a false representation of the general distribution of the stars. So the

exceedingly high density he found for the most southern part of the Galaxy, in the Southern Cross, which makes these regions the richest of the sky, is not real, but chiefly a consequence of the great density and the low limiting magnitude of charts 50 and 55. In the same way most of the conclusions based on his numbers must be revised.

3. By application of the corrections here found, however, we obtain values for the star density that are fairly homogeneous over the whole sky. They are therefore the best material we possess at this moment to study the distribution of these stars, better indeed than the catalogue plates of the Carte du Ciel, that extend to a somewhat lower limiting magnitude.

For this purpose each chart has been divided into four quadrants (of 15° square), and for every quadrant the mean corrected density (usually from 25 fields) was computed. For the centre of each quadrant the galactic longitude and latitude were read from the abacus in Dr. Nort's memoir (a rather rough method, as errors of 1° or perhaps more may occur, but for our purpose it is sufficient). In Table II. is given a catalogue of these mean densities; the first two columns give the galactic co-ordinates of the centre, the third column gives the number of the chart, the fourth the star density per square degree.

TABLE II.

Mean Star Densities arranged in order of Galactic Longitudes.

Galactic Long.	Galactic Lat.	Chart Number.	Stars per Sq. Deg.	Logarithmic Deviation, unit 0·01.	Galactic Long.	Galactic Lat.	Chart Number.	Stars per Sq. Deg.	Logarithmic Deviation, unit 0·01.
1°	−59°	45	15·4	+15	34°	+64°	17	10·8	+02
2	+53	18	13·3	+05	35	+18	19	37·7	+12
6	+ 6	31	29·6	−09	39	+11	20	62·2	+26
6	−28	32	25·9	+08	41	−22	21	37·0	+16
8	+38	18	22·6	+14	42	−63	22	14·7	+15
12	+26	19	30·4	+13	43	−43	33	18·5	+11
12	−72	34	12·2	+12	45	+36	8	25·9	+18
13	− 6	32	43·3	+07	47	+47	7	17·9	+13
13	−40	33	23·6	+19	47	− 3	20	59·5	+20
19	+12	19	44·0	+12	48	+19	8	28·3	+01
21	−20	32	34·1	+10	52	− 8	21	52·0	+16
24	0	20	43·0	+05	53	+ 8	9	50·5	+15
26	−54	33	17·1	+17	54	−32	21	31·6	+22
29	−30	33	24·9	+09	59	−51	22	13·1	+03
30	+56	18	12·1	+03	60	−85	34	10·6	+09
30	+40	18	19·9	+11	61	+76	17	10·3	+06
30	+33	19	22·2	+08	62	+62	7	13·0	+09
31	−10	20	38·3	+04	64	− 3	9	88·6	+37

TABLE II.—*continued.*

Galactic Long.	Galactic Lat.	Chart Number.	Stars per Sq. Deg.	Logarithmic Deviation, unit 0·01.
65°	−18°	21	32·9	+06
67	+38	7	17·5	+03
67	+37	8	15·8	−02
67	+21	8	28·1	+03
67	+20	9	31·9	+07
67	−37	10	19·4	+07
73	−21	10	33·1	+10
75	− 9	2	43·2	+09
77	+ 8	9	46·6	+11
78	+ 7	2	41·4	+05
80	+20	1	18·9	−16
82	+48	7	10·7	−08
82	+37	1	13·0	−11
82	−71	22	13·2	+15
84	+48	6	11·2	−06
84	+ 5	2	34·4	−04
85	+66	6	10·3	+01
86	+ 6	3	37·9	+01
86	−55	22	11·0	−02
87	−41	10	14·3	−02
89	−23	10	20·3	−09
91	−12	2	26·8	−10
94	+78	16	9·2	+01
96	+17	1	32·5	+04
98	−23	11	23·7	−02
102	+34	1	14·7	−09
103	−40	11	13·1	−07
107	+45	6	9·9	14
108	+44	5	11·5	−09
108	+13	3	22·8	−16
108	−53	23	14·7	+09
109	+14	4	25·5	−10
113	−10	3	40·1	+06
113	−18	11	19·7	−16
115	+30	5	12·6	−20
115	+28	4	17·3	−09
118	− 2	3	36·6	−02
120	+60	6	10·4	−02
120	−14°	12	17·3	−27
120	−34	11	9·5	−28
120	−68	23	13·4	+14
126	+ 7	4	27·2	−13
131	−46	23	16·3	+08
132	+50	5	7·2	−24
132	− 3	12	18·4	−31
133	+21	4	22·0	−08
133	−26	12	10·6	−33
136	+34	5	10·2	−25
137	+ 2	13	24·8	−18
140	+68	16	10·0	+01
144	+16	13	15·9	−28
144	−16	12	9·3	−51
145	−37	24	15·4	−03
147	+62	15	10·1	−02
147	+23	14	12·7	−29
150	+46	15	9·5	−15
150	+38	14	11·2	−16
150	−58	23	12·4	+05
152	− 6	13	21·5	−23
156	−24	24	12·0	−30
158	+ 7	13	25·9	−15
158	−79	35	8·7	−01
162	−47	24	13·3	00
164	+19	14	17·3	−21
164	−13	25	19·0	−23
170	+33	14	13·5	−14
171	+ 1	25	28·7	−12
172	−33	24	21·3	+06
175	+45	15	13·7	00
176	−65	35	11·8	+07
177	+13	26	32·5	00
179	−20	25	18·3	−17
181	+59	15	11·0	00
182	−26	37	12·6	−25
182	−53	36	17·4	+17
186	+28	26	15·3	−14

TABLE II.—*continued.*

Galactic Long.	Galactic Lat.	Chart Number.	Stars per Sq. Deg.	Logarithmic Deviation, unit 0·01.	Galactic Long.	Galactic Lat.	Chart Number.	Stars per Sq. Deg.	Logarithmic Deviation, unit 0·01.
186°	− 6°	25	34·8	−02	253°	+21°	40	28·0	+02
188	−38	36	13·5	−08	255	+ 9	50	25·1	−15
192	+72	16	10·7	+06	257	− 8	49	51·5	+16
193	+40	27	17·7	+06	258	− 7	50	63·0	+24
193	+ 6	26	35·8	−01	262	+71	28	15·6	+22
199	−12	37	25·8	−11	262	−25	54	28·6	+09
201	+20	26	20·2	−13	262	−48	47	12·1	−03
204	0	38	28·8	−12	263	−42	54	12·3	−08
206	+54	27	14·4	+09	264	− 5	50	64·5	+24
209	+30	27	19·2	−02	264	−48	46	14·8	+06
210	−33	37	13·7	−13	265	−15	55	21·1	−17
210	−40	36	18·5	+08	265	−66	46	13·4	+13
210	−56	36	19·3	+23	266	+55	28	13·9	+08
212	+12	38	29·0	−06	266	− 6	51	61·0	+22
214	−64	35	15·9	+19	267	+41	40	15·6	+02
215	−18	37	23·3	−09	269	+23	40	25·1	00
219	−11	38	19·3	−25	270	− 3	55	59·1	+20
221	+22	39	27·7	+03	271	+12	50	26·6	−10
222	+63	28	16·5	+21	274	−78	34	11·4	+11
223	+43	27	16·5	+06	277	−24	54	35·9	+17
225	−36	48	18·3	+03	278	+23	41	24·1	−01
227	+ 3	38	46·5	+09	280	−23	55	40·0	+21
227	−47	47	15·9	+08	282	−37	54	17·0	+01
228	−19	48	19·9	−15	283	+40	41	16·2	+02
232	+ 8	39	35·0	−01	284	− 8	55	53·2	+17
233	− 8	49	37·2	+02	287	−45	46	13·9	+01
234	+32	39	23·0	+08	288	+53	29	17·5	+17
239	+51	28	12·2	00	288	−13	51	43·2	+12
240	+85	16	9·0	+02	288	−44	53	15·8	+05
241	−76	35	11·1	+09	289	−14	52	35·2	+04
242	−62	47	15·1	+16	293	+18	41	31·1	+03
244	+ 3	49	44·6	+07	293	+10	51	51·7	+17
245	+18	39	37·1	+11	295	−28	52	28·6	+13
247	+37	40	17·6	+03	295	−30	53	19·6	−01
247	−20	49	21·5	−10	298	+ 2	51	45·7	+08
247	−21	48	20·5	−11	300	+68	29	16·1	+22
247	−37	48	18·7	+05	300	+34	41	15·7	−06
247	−38	47	15·1	−03	300	+14	42	27·3	−07

TABLE II.—*continued.*

Galactic Long.	Galactic Lat.	Chart Number.	Stars per Sq. Deg.	Logarithmic Deviation, unit 0·01.	Galactic Long.	Galactic Lat.	Chart Number.	Stars per Sq. Deg.	Logarithmic Deviation, unit 0·01.
300°	−60°	46	13·4	+09	330°	−38°	44	14·0	−06
306	− 7	52	44·9	+09	330	−46	45	16·3	+08
311	+46	29	16·0	+08	332	+ 6	43	28·8	−11
312	+ 3	42	30·6	−09	336	+24	30	10·1	−38
312	−50	53	11·5	−03	338	+79	17	9·1	+01
313	+26	42	18·0	−10	338	− 7	43	25·4	−16
313	−21	52	33·5	+10	342	+47	30	14·3	−04
316	−34	53	20·7	+06	344	+13	31	17·2	−28
317	− 2	43	36·4	−02	344	−19	44	17·7	−20
320	−68	34	16·1	+22	350	−33	44	18 5	00
324	+16	42	17·2	−25	351	− 1	31	29·8	−11
324	−16	43	25·3	−08	352	+33	30	15·2	−08
325	+37	30	14·0	−08	355	−45	45	18·7	+13
327	−23	44	10·6	−37	356	+65	17	10·9	+03
327	−62	45	14·2	+13	357	−13	32	19·8	−22
330	+58	29	14·8	+13	359	+20	31	20·2	−13

4. The distribution of the star densities over the sky may be studied in two different ways: by expressing it by numerical formulæ — this is the way followed by Mr. Plummer — or by plotting them on a map. In the first case we try to express the density as a function of galactic latitude and longitude. To find the decrease with galactic latitude the logarithms of the densities were condensed into the following mean values :—

Mean Lat.	Log N.	*n*.	*m*.
2°·6	1·594	18	11·42
7·5	1·587	25	11·46
12·8	1·429	18	11·18
19·8	1·370	37	11·23
29·8	1·250	29	11·24
39·1	1·209	29	11·40
49·1	1·132	28	11·41
61·0	1·121	18	11·65
73·7	1·059	18	11·65

Using the table of Van Rhijn,* we find from these log N the limiting magnitudes given in the fourth column; their weighted

* Dr. P. J. van Rhijn, "On the Number of Stars of each Photographic Magnitude in different Galactic Latitudes," table iv. (*Public. Groningen*, No. 27, p. 62).

mean 11·38 may be considered as the limiting photographic magnitude belonging to the Harvard Map reduced in this manner.

The dependence of density on galactic longitude has a meaning for the lower galactic latitudes only. For different zones between $\pm 40^\circ$ the densities were plotted in a diagram and curves were drawn through them. They all show the same character: two maxima of density in the first and third, two minima in the second and fourth quadrant of longitude. They may be represented by the formulæ—

$+40^\circ$ to $+21^\circ(32)$	$\log N =$	1·245 + ·028 cos λ	− ·045 sin λ	− ·053 cos 2λ	+ ·130 sin 2λ	− 000 cos 3λ	+ ·030 sin 3λ
+20 ,, +11(20)		1·424 + ·020	− ·020	− ·048	+ ·134	− ·020	+ ·028
+10 ,, −10(43)		1·575 + ·052	− ·035	− ·117	+ ·075	− 027	+ ·015
−11 ,, −20(19)		1·374 + ·099	− ·055	− ·117	+ ·082	− 053	+ ·018
−21 ,, −40(34)		1·285 + ·099	− ·032	− ·077	+ ·107	− 037	+ ·053

The terms of the first and second order may also be written—

1·245 + ·053 cos $(\lambda - 302^\circ)$	+ ·140 cos $2(\lambda - 56^\circ)$		
1·424 + ·027	315	·142	55
1·575 + ·063	326	·139	74
1·374 + ·113	331	·143	72
1·285 + ·104	342	·132	63

All these zones show a great resemblance in their formulæ. If we take their weighted mean, we obtain a formula that represents the general distribution of the 11th magnitude stars along the galactic zone.

$$\log N = 1·389 + ·059 \cos\lambda - ·037 \sin\lambda - ·085 \cos 2\lambda + ·103 \sin 2\lambda - ·026 \cos 3\lambda + ·029 \sin 3\lambda$$
$$= 1·389 + ·070 \cos(\lambda - 328^\circ) + ·134 \cos 2(\lambda - 65^\circ) + ·039 \cos 3(\lambda - 44^\circ)$$

The first term may be explained by an eccentric place of the sun in this star system (the centre of the system lying towards $\lambda = 328^\circ$). The second term means a greater density along the axis $65^\circ - 245^\circ$, a smaller density in a direction perpendicular to it, thus making the figure of our star system a three-axial ellipsoid with densities 8·6, 18, and 33 along the three axes. The higher terms are small; they may be real—the coefficients of cos 3λ and sin 3λ have the same sign in all the zones—but they have no physical meaning: the excess of stars at $\lambda = 44^\circ$, 164°, and 284°, and the lack of stars at $\lambda 104^\circ$, 224°, and 344°, all together expressed by the third term, have no organic connection with each other. The irregularities these higher terms represent are local; they are connected with the star clouds and the void spaces of the Milky Way itself, and therefore they are not aptly represented by a formula. Perhaps the same may be said of the first term, as it expresses only the fact that the minimum density at $\lambda 150^\circ$ is lower than the minimum at $\lambda 330^\circ$.

The results here found are very different from Mr. Plummer's; this is not astonishing, as the foundations of his calculations need large corrections. We have not pursued further our research in this direction, because periodic functions, spherical harmonics, or other formulæ are inadequate to represent the irregularities of the star distribution.

5. The special features of the distribution of the stars show themselves at once when we plot the densities on a map or a globe. Next to the general decrease from the Milky Way to its poles, we detect by this method immediately, by mere inspection, all the irregularities in the distribution. Two regions of greatest density in the galactic zone appear, nearly opposite to each other, one in Cygnus and Lacerta, the other extending from α and β Centauri, on the southern side of the Cross, to β and ϵ Carinæ. Between them we find the poorest parts of the Milky Way in Auriga and Gemini, and on the other side between Aquila and the sting of the Scorpion. This distribution of the stars to the 11th magnitude does not correspond to the distribution of light in the Milky Way; this light is feeblest in Perseus and becomes brighter in Auriga, and on the other side the bright patches in Aquila, Scutum, and Sagittarius shine through the thin layer of 11th magnitude stars. Where the density in the galactic zone is least the void spaces around the galactic poles reach farthest to the Milky Way; on the northern side a region with density < 10 extends from Arcturus to the forelegs of Ursa Major, and on the southern side a most curious isolated void space lies in Taurus and Aries immediately on the western border of the Milky Way. On the opposite side, in Sagittarius and Ophiuchus, this phenomenon is less clearly indicated.

We get a still better picture of the peculiarities in the distribution of the stars if we eliminate the decrease to the galactic poles by comparing the observed densities not with an equal distribution but with a spheroidal star system, where the density depends only on magnitude and galactic latitude. With the limiting magnitude 11·38 we take from Van Rhijn's table iv. the normal density (log N) for each latitude; the deviations of the logarithms of the real densities from these normal log N are inserted in Table II. in the last column. The distribution of these numbers over the sky show the anomalies in the real distribution of the stars, compared with the typical oblate star system.

If we plot them on a globe and draw isanomalous lines (see fig. 1) we find two opposite regions of deficiency, one greater, extending from the celestial North Pole over Orion and Argo to the South Pole, the other smaller, covering Ophiuchus, Aquila, Sagittarius, Scorpio, and Lupus. They are separated by a zone of surplus density; in it the regions of greatest excess lie, of course, in the Milky Way, one in Cygnus-Lacerta, the other on the other side, south of the Cross and in Musca. Other less-marked regions of excess lie in Virgo, near Fomalhaut, and in the southern part of Eridanus. In each of the regions of deficiency lies a centre of

greatest defect, one in Taurus ($\alpha = 65°$, $\delta = +25°$), the other in Ophiuchus ($\alpha = 250°$, $\delta = -10°$), nearly opposite to each other. The first is the remarkable region at the border of the Milky Way,

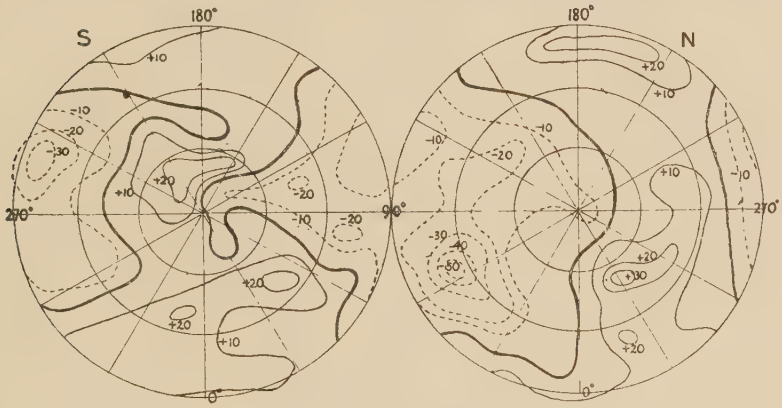

Fig. 1.

already noted above, where the density is only $\frac{1}{3}$ of what it should be in a regular universe; the other is somewhat less deep.* They appear like funnel-shaped pits in the flattened star mass. If we represent the distribution of these stars by the irregular surface

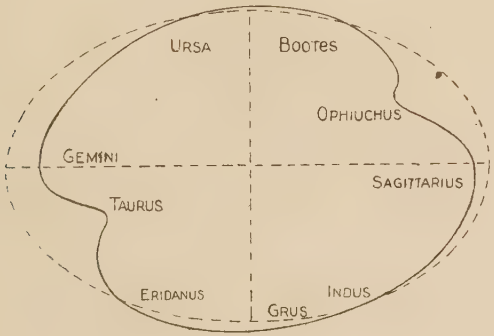

Fig. 2.

of a star cloud with equal volume-density throughout—thus making each radius proportional to the cube root of the number of stars,—its cross-section takes the shape of fig. 2.

* Both are also clearly indicated on the Catalogue plates of the Carte du Ciel, *cf.* Frederick Seares, "The Galactic Condensation," table vi. and fig. 2 (*Ap. J.*, **46**, 131–133). From a separate count we found afterwards the centre of the first to be at $\alpha = 68°$ $\delta = +27°$, with a star density only one-eighth of the normal density.

It is clear that this distribution cannot in any way be represented by a three-axial ellipsoid; the deviations from the oblate star spheroid bear quite another character. We need not take account here of the maxima of excess, for they are local features and are connected with the remote star clouds of the Milky Way itself. After the oblateness along the galactic plane as the first great deviation from equal distribution there remain, as the second systematic deviation, the two opposite regions of paucity, that arrange themselves around the funnel-shaped pits in Taurus and Ophiuchus as their cores. From these cores the lack of stars spreads itself wide around, and causes the small star density in the neighbouring parts of the Milky Way, which appeared in the formulæ above as the terms with 2λ. Their influence is not symmetrical on all sides: the variation is steep and sudden at

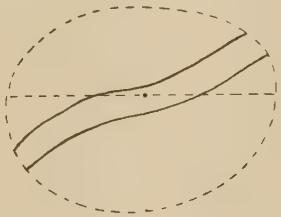

FIG. 3.

the western side, away from the Milky Way; on the other side it is gradual, and extends far across the Milky Way.

6. To explain this remarkable anomaly in the regular distribution of the stars, we may connect the Taurus pit with the nebulosities revealed by photography in the region of the Pleiades. We must then assume that in Taurus extensive dark nebulous matter, of which the brightest luminous parts are visible around the stars of the Pleiades, obscures the far removed stars; these absorbing masses must stretch themselves, gradually thinning out, across the Milky Way, over a great part of the sky from pole to pole, thus causing a general lack of stars in these regions. Then we must assume the same for the Ophiuchus pit, and perhaps the remarkable bright and dark nebulosities photographed by Barnard in Scorpio are the densest outlying parts of this nebulous matter. In this explanation it is mere accident that the two strongest darkened regions lie opposite to each other; they have no physical connection. Another difficulty lies in the fact that we find the brightest clouds and patches of the Milky Way just in the regions of Scutum and Sagittarius that should be obscured by near absorbing nebulosity.

Another explanation may be found in the supposition that a dark lane, a channel with small star density, pierces the star cloud around our sun in the direction of these pits. Then we will see a lack of stars in two opposite directions. By assuming

the channel somewhat curved and the sun out of its axis (as in fig. 3), we may explain why the funnels lie unsymmetrically in the regions of deficiency and are not exactly opposite to each other. But it is useless to go into such details before the results of the Harvard Map are either confirmed or corrected by other data.

Summary.

1. The star densities given by Dr. Nort in his memoir and used by Mr. Plummer are not homogeneous.

2. By multiplying them by the coefficients given in Table I. they are reduced to a homogeneous system that affords the best material now available to study the distribution of the stars to the 11th magnitude.

3. Next to the regular decrease with galactic latitude the mathematical analysis shows a variation with the galactic longitude, especially a term with 2λ, which indicates a three-axial ellipsoid as the figure of our star system.

4. By plotting the logarithmic deviations from a star density regularly decreasing with galactic latitude, the cause of this variation becomes manifest. Two regions of deficiency are shown, arranged around two opposite spots in Taurus and Ophiuchus, where the star density is only $\frac{1}{3}$ of the normal density or still less. They may be explained by extensive absorbing nebulosities or by a dark channel traversing our star system.

Preliminary Note on the Application of Photoelectric Photometry to Astronomy. By A. F. and F. A. Lindemann.

Some ten years ago Elster and Geitel suggested that photoelectric phenomena would form a useful means of comparing the light received from different sources. This suggestion was first put into practice for astronomical purposes by Rosenhain and Meyer at Tübingen, and by Guthnick at the Babelsberg Observatory. Since no details of their work are to hand, a description of the methods which had been developed successfully at the Sidholme Observatory may prove of interest to astronomers, even though the weather has prevented the taking of more than a few preliminary observations. There are a number of problems of considerable interest which cannot be adequately studied by the usual photometric arrangements. The photoelectric method combines comparatively high accuracy with the advantage of being applicable to most diverse conditions. Since it does not necessitate a very good lens or mirror or a very accurate clock, it is hoped that it will be widely adopted once its simplicity and reliability are realised.

General Principles of Photoelectric Photometry.

Light falling upon a metal surface causes the metal to emit electrons. The energy of these electrons $eV = h(\nu - \nu_0)$, e being the elementary charge $4{\cdot}77 \cdot 10^{-10}$ E.S.U., V the potential in electrostatic units, h Planck's constant $6{\cdot}55 \cdot 10^{-27}$, ν the frequency of the light, and ν_0 a frequency characteristic of the metal. ν_0 is larger the more electronegative the metal, and since it corresponds to ultra-violet light for all metals except the alkalis and alkaline earths, it is usually advisable to use as electropositive a metal as is convenient.

The number of electrons emitted per unit incident energy varies with the wave-lengths, and depends upon the state of the surface. The so-called normal photoelectric current, which is independent of the plane of polarisation and angle of incidence, is insignificant in the visible spectrum. The alkali metals, however, show a well-marked "selective" effect for other than normal incidence, which usually amounts to some hundreds of times the current obtained from the normal effect. The selective current is produced by light of wave-length 3700 Å. to 5000 Å., with a maximum at 4400 Å. if potassium is used. With rubidium the maximum is at 4800 Å., with cæsium probably at about 5500 Å. If suitable surfaces free from oxide are used, some 2 per cent. of the incident light energy reappears in the form of electron emission.

The current produced is proportional to the amount of the incident light, and is entirely independent of the intensity per unit surface. It is this property which renders photoelectric photometry so valuable for measuring in absolute units the light received from objects like comets and nebulæ.

Like the eye or the photographic plate, the photoelectric cell is confined to a relatively small part of the spectrum, as the normal effect is outweighed by the selective effect. Since this band is narrower and more sharply defined than in either of the above instruments, this defect may be converted into an absolute advantage if two cells containing different metals are employed (see below).

The principle of the method adopted consists in measuring the number of electrons emitted by the photoelectric metal, which is placed close behind the focus of the telescope, so that the light from the object to be measured falls upon it. The ratio of this number to the number emitted under the influence of a standard source of light is then a measure of the energy received.

Since the energy received per cm.^{2} from a star of magnitude 0 is of the order of 10^{-5} ergs per second, and only a small part of the light affects the photoelectric cell, where again not more than 2 per cent. becomes available as current, every precaution must be taken to use sensitive methods of measurement. For this reason it is advisable to increase the original photoelectric current by enabling each electron to form new ions by collision.

This may be done by surrounding the sensitive metal with a

neutral gas and applying a negative potential to the metal. It may be shown that the optimum pressure of the gas is given by the condition that the potential drop along the free path is equal to the effective ionisation potential. Since the total applied voltage is limited to about 150 volts by the fact that the luminous discharge must be avoided, the pressure required is that at which the free path is equal to the distance between the electrodes multiplied by the effective ionisation voltage over 150. At this pressure each electron forms $\epsilon^{\frac{v}{Ve}}$ ions, v being the applied voltage and V the effective ionisation potential, so that a considerable magnification of the original photoelectric current may be obtained.

Description of Photoelectric Cell and Method of Filling.

As was pointed out above for the photometry of faint visible light, one is practically confined to the metals potassium, rubidium, and cæsium. The use of these entails the use of one of the rare gases. Though it is hoped to use rubidium and cæsium cells shortly, all the preliminary experiments have been carried out with potassium. The form of the cells used is shown in the diagram (fig. 1).

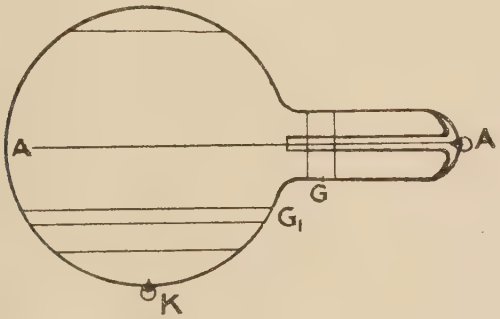

Fig. 1.

K is the photoelectric metal which forms the cathode, A the anode, consisting of a ring of platinum wire supported by a thin glass tube. At G a thin platinum wire passes through the wall of the tube, which is silvered inside and surrounded by tinfoil outside. This is earthed, thus forming a double guard-ring, which is usually supplemented by a further external guard-ring at G_1. The bottom of the bulb surrounding the platinum wire K is silvered to ensure a good contact with the potassium. The method of preparing the cells is shown in fig. 2.

The potassium is cleaned and freed from paraffin as far as possible and introduced into the tube T, which is then sealed off at 1. The whole apparatus is then evacuated to as high a vacuum as possible with a Gaede mercury pump, which is kept rotating for

the whole of the rest of the operation. The potassium is melted, and runs down into the bulb B_1 through the three inverted funnels F_1, F_2, F_3. The openings of these funnels are about 1 millimetre wide, so that most of the outer skin of the potassium which contains the paraffin remains in this part. The tube T with the funnels is then sealed off from the rest of the apparatus at 2, and the potassium distilled successively into B_2 and B_3, being sealed off at 3 and 4. The potassium is then melted, and the joint at J turned until it runs down into the bottom of the cell so that it covers the cathode K. It is preferable to distil a little potassium

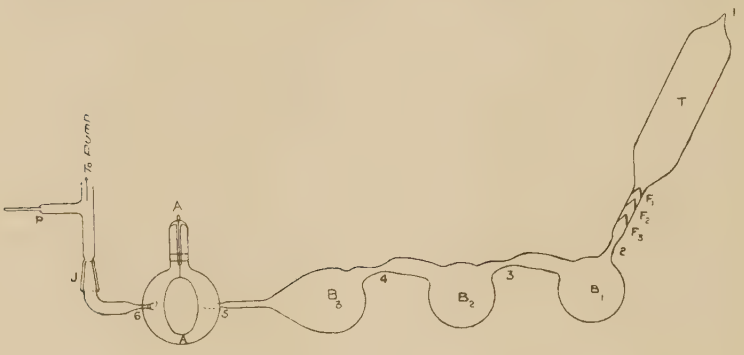

FIG. 2.

over the walls of the cell with the exception of a circle of 2 cm. in diameter, through which the light falls into the cathode. Care must be taken to keep the anode tube and its immediate neighbourhood warm during this operation to prevent any potassium condensing there. The most convenient way to do this is by means of a small electric heater. Unless it can be done quickly this operation is best not done at all, since the potassium vapour rapidly reduces the silica in the glass, thus spoiling the insulation.

The last bulb is sealed off at 5, and the potassium must next be treated to render the surface absorbent. For this purpose the tap to the pump is turned off and pure hydrogen is introduced into the cell at a pressure of 0·5 to 1·0 millimetre of mercury, which may be effected conveniently by means of a palladium tube P welded up at one end and welded on to a platinum tube which is sealed into a glass tube at the other. The cathode K is connected to the negative terminal of a small induction coil ($\frac{1}{2}$-inch spark), and a discharge of a few microamperes sent through until the potassium surface becomes a deep violet in colour, probably owing to the formation of colloidal potassium. The hydrogen is then pumped out and helium introduced until a pressure of about 0·75 millimetre is reached. The cell is then sealed off at 6 and is ready for use.

Rubidium and cæsium cells may be prepared in an exactly similar way, except that the metal is usually too scarce to permit

the loss in the inverted funnels. These metals are therefore washed in petrol, and dried in a current of pure hydrogen or nitrogen in the first bulb. The finished rubidium cell is green in colour, the cæsium yellow.*

Method of Measuring Photoelectric Current.

If one takes the proportion of light effective as 1/10 and assumes an efficiency of 2 per cent. one would expect a photoelectric current of the order of 2.10^{-13} ampere from a star of magnitude 0 using a 6-inch telescope. Even using ionisation by collision, this corresponds to less than 10^{-11} amperes, so that stellar photometry requires an electrometer. There are two methods which can be used. The first is to connect the anode to the electrometer and to measure the rate at which its potential increases. If the capacity is constant, this is obviously a measure of the current, *i.e.* of the incident light. The other method is to connect the anode to earth through a small constant leak, *i.e.* through a resistance of the order of some 10^{13} ohms. When a current flows from the potassium to the anode the potential of the anode will rise until the amount of electricity escaping through the leak balances the inflowing current. This potential defines the current if the resistance of the leak is constant. The first method is clearly the more sensitive, and this was therefore usually employed.

For this purpose it is of course essential that the capacity be constant, and as this may change by mutual induction if any of the parts change their relative positions, it was determined to fix the electrometer rigidly to the cell, which again is fixed to the telescope. This necessitates the use of a string electrometer, and since no suitable instrument could be procured, a special form, characterised by lightness and simplicity, was designed and constructed.

Description of Electrometer.

The details of the electrometer and of the mounting of the cell may best be seen by referring to the detailed drawing (fig. 3, Plate 4). The electrometer is of the single-fibre Einthoven type, the potential being measured by the movement of a thin silvered glass or silica fibre (F) connected to the anode (A) in the electric field of two parallel plates kept at a constant high difference of potential by means of the dry piles (P_1 and P_2), or if preferred by accumulators, the centre of course being earthed in either case. The fibre is between $\frac{1}{2000}$ and $\frac{1}{4000}$ of a millimetre thick, and is mounted on a frame (F_1, F_2) of the same material as itself, made of rod about 3 millimetres thick. This frame is about 100 millimetres long, and is made, as shown, with a ring (R) in the middle to allow the fibre to be illuminated. At one end (F_1) it is bent up about 12

* We are much indebted to Mr. Keeley of the R.A.E. for his help in making many of our cells.

millimetres and a slit sawn into it, in which the fibre and the connecting wire are secured with Wood's metal. The other end of the fibre may be secured either to a rotating axle (*c*), which can be turned from outside, as shown in the drawing (3*b*), or to a thin glass or silica spring, which can be bent sufficiently to give the fibre any required sensitivity by making it tauter or slacker, as shown in (3*c*). The frame is mounted on ebonite in a brass box. If the end (F_2) is secured to the box, and the regulating screw is made of the same metal as the box, no change of sensitiveness with temperature need be apprehended. Using glass, there is a change while the electrometer is being heated or cooled, as the fibre takes up the temperature of the surrounding air much more quickly than the frame.

On either side of the fibre, 4 millimetres apart, mounted on ebonite, are the two plates, or rather bars (D_1, D_2), 4 millimetres by 4 millimetres by 100 millimetres, which can be moved laterally so as to adjust the fibre exactly half-way between them. Stout copper wires carried through ebonite plugs in the walls of the box allow them to be charged to any desired potential. All ebonite parts inside the box are covered with shellac.

In one end of the box is a small glass vessel (V), which is filled with sodium in order to keep the inside of the electrometer dry. This is covered with a metal cap, both to protect it and to keep out the light. A tap (T) allows the sodium to be changed without letting moist air into the box.

The movement of the fibre is observed with a microscope (M), and measured on a scale in the eyepiece in the ordinary way. For many laboratory purposes it is convenient to close the holes in the top and bottom of the electrometer with a thin glass plate. The whole electrometer is then about 13 centimetres long by 4·5 centimetres broad by 2·8 centimetres deep, weighs about 500 grs., and can be placed upon the stage of any ordinary microscope. The sensitivity can be varied within wide limits, something like 100 divisions per volt being attainable if an objective of fair working distance is available and an eyepiece of reasonably high magnification is used.

For astronomical use, however, the microscope was screwed on to the electrometer box, adjustable laterally, and artificial illumination arranged. Since no objective with a long working distance was immediately to be had, a $\frac{1}{4}$-inch objective was used, an air-tight joint, which yet permits motion of the parts, being made by means of indiarubber. The magnification with the eyepiece used was 240.

A fitting for recording the movement of the fibre photographically on a moving cinematograph film has been constructed, but has not been used yet.

The only point to be mentioned about the artificial illumination is that a deep red glass was inserted between the lamp and the electrometer box. Any stray light from the lamp is thus less likely to affect the potassium, though not perhaps the cæsium, cell,

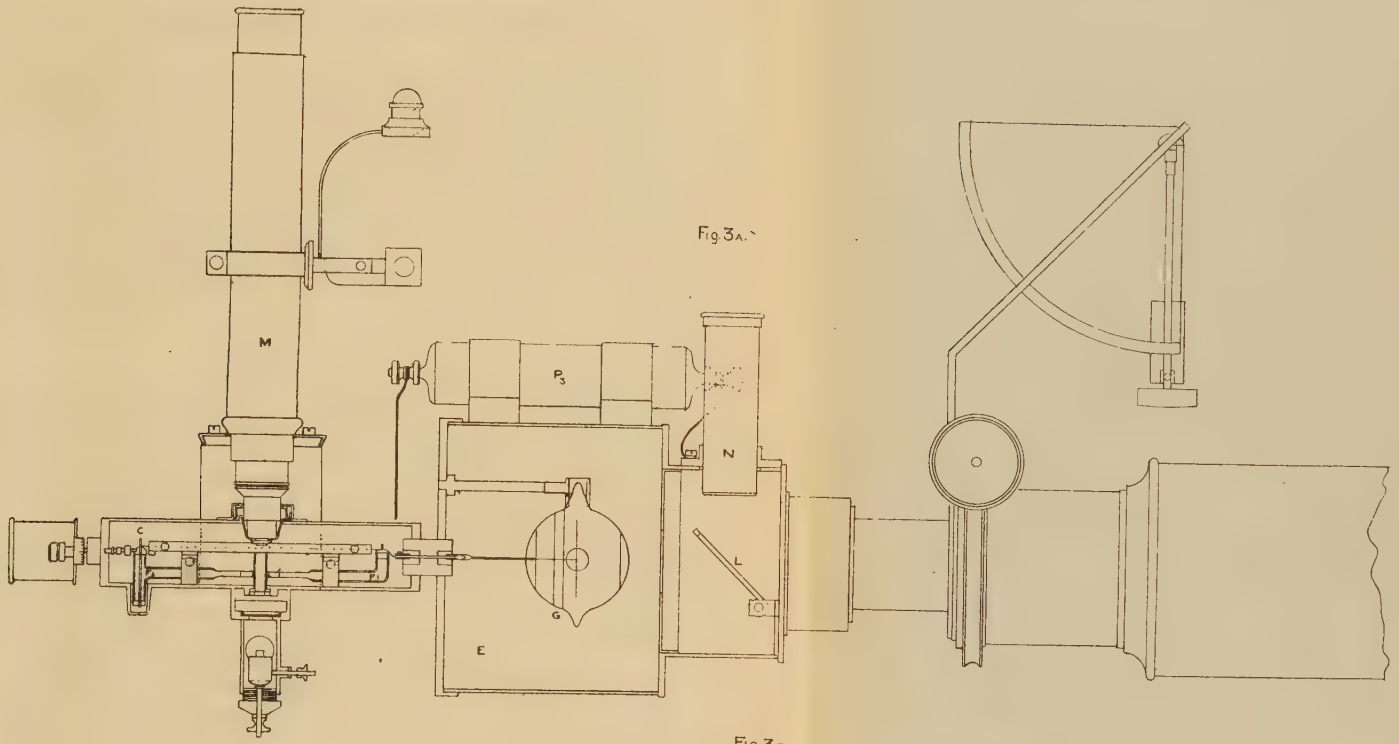

Fig. 3A.

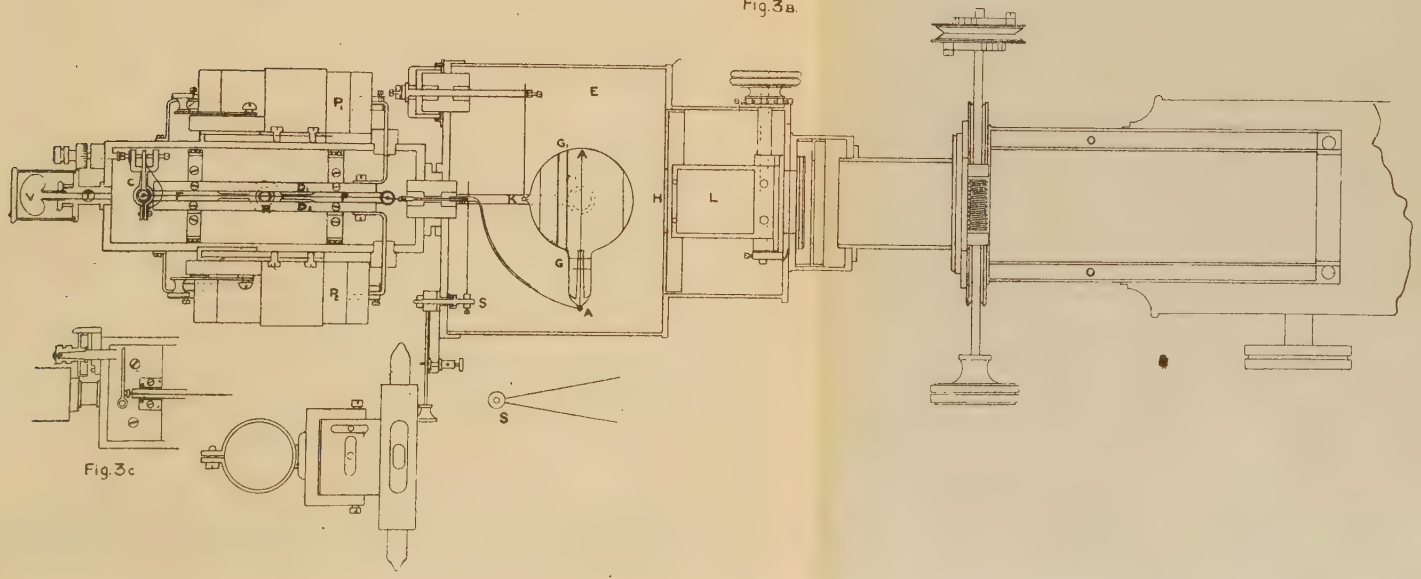

Fig. 3B.

Fig. 3C

especially as accurate reading is possible with a weaker light if this be red.

It is often convenient to use dry piles for the high potential on the plates. These were fixed as shown in the drawing, and consisted of copper, copper oxide, zinc sulphate, and zinc separated by discs of paraffined paper through a slit in which a strip of tinfoil made an electric connection. They were pressed together in a paraffined glass or silica tube and hermetically closed, and have given satisfactory results up to date, but whether they will maintain their voltage it is as yet too early to say. So far, at the end of six weeks, they appear to give about 1 volt per cell, or 70 volts each set.

Mounting of Cell.

The wire connecting the electrometer fibre to the photoelectric cell is carried by an amber plug through the end wall of the electrometer box into the cylindrical box (E) in which the cell is mounted, as shown in the drawing. A switch (S) enables the electrometer fibre to be connected to earth or to a standard voltage if desired. The accelerating potential applied to the cathode may again be taken from a dry pile (P_3) as shown, the positive pole being earthed. The guard-ring (G) on the cell as well as the second external guard-ring (G_1) are earthed. The cell is held either by one of the sealed off-side tubes or by the main guard-ring. A hole about 25 millimetres in diameter is covered by a thin plane glass (H), through which the light from the star enters the cell box.

Between this and the focus is a mirror (L), which may be turned through 45°, so as to throw the light at the focus into the eyepiece (N) or to allow it to pass into the cell. At the focus is an iris diaphragm with a divided scale, which can be opened to 25 millimetres and closed down to 0·5 millimetre. With this arrangement any desired object may be set and extraneous light cut out.

Since the zero of the electrometer shifts if it is tilted sideways, various automatic arrangements were used for keeping it horizontal about an axis transverse to the fibre. The whole was able to rotate about the tube (O), which was fixed to the telescope, and an electric release allowed a clock to turn the instrument about the axis of the telescope when it was out of level. In practice it was found sufficient to level the electrometer by hand before reading. A quadrant enables the height of the star to be read directly, using the pendulum of the automatic regulator.

A glance at the drawing will show that cell and electrometer are rigidly fixed together, almost entirely enclosed in an earthed metal box, so that no change of capacity need be apprehended whatever the position of the telescope.

Procedure in making a Photometric Measurement.

Though the stellar magnitudes should be compared to a standard source of light and given in absolute units, it is difficult to avoid

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Procedure in making a Photometric Measurement.

Though the stellar magnitudes should be compared to a standard source of light and given in absolute units, it is difficult to avoid

complications due to the different temperatures. For the present, therefore, all magnitudes measured have been comparative only. As in all photometric determinations, relative measurements are usually to be recommended on account of the varying opacity of the air. The correction of the height may of course be found in the usual way.

Measurements were usually carried out in the following manner. All connections were made, and the instrument allowed to remain in the dark for about an hour. There was often a considerable shift of the zero for the first half hour, possibly owing to induction from charges, which gradually accumulated on parts of the glass cell walls, or more probably due to the settling down of the accumulators, which were sometimes used. The cell was then tested for leak, *i.e.* the electrometer observed for, say, ten minutes. The leak in ten minutes was seldom more than one division, *i.e.* less than 0·02 volt or about 4.10^{-16} amperes.

The telescope was now set on the desired object, the electrometer levelled, the mirror put into position for observing through the eyepiece, and the iris closed until all extraneous light was cut off. The mirror was then turned until the light could reach the cell and a stop-watch started. After an appropriate interval, depending on the luminosity of the object to be measured, the light was again cut off by turning the mirror and the stop-watch stopped. The electrometer was levelled and read and the fibre earthed and read, the clock being thrown out of gear if this operation took more than a minute or so. The standard voltage was now switched on and the electrometer read, which completed the measurement.

Exactly the same procedure was gone through with some adjacent standard star. If the object to be measured charged the electrometer at, say, X volts per second and the standard star at A volts per second, the light received from the object is X/A times the light received from the standard star.

Some Applications of Photoelectric Photometry.

Only a few preliminary trials have been carried out, as the weather during the last four months has prevented any systematic work, even the few fine nights at the end of 1918 December being marred by auroræ for work on nebulæ. Still the results appear sufficiently promising to warrant the publication of this paper. The applications of the method are so numerous that it is hoped that other observatories which enjoy the advantage of professional assistance in making the instruments may take up the question seriously.

A few of these applications will be discussed in the following pages, and illustrated wherever possible by the preliminary observations.

Stellar Magnitudes.

Even though the following figures were simply the result of preliminary tests, they show that there is every prospect of attaining the same accuracy, comparing stellar magnitudes with the

instrument described above, as is claimed by the German observers. This is shown by a series of six sets of observations of Capella given in full. The readings are scale divisions, 39 of which are equal to 1 volt. Light wisps of cloud which had been present all day are indicated unmistakably in series 1 and 3.

4.3.1919. S. T. 9^h to 9^h 30^m. α Aurigæ (G). $h = 51°$. 120 volts accelerating potential. 39 div. = 1 volt.

Divisions.	Time.	Time.	Time.	Time.	Time.	Time.
5	8·2	7·4	8·4	7·8	8·0	7·2
10	17·8	15·8	16·6	15·6	16·0	15·8
15	27·0	24·6	25·4	24·6	24·0	24·6
20	38·0	34·2	36·0	34·4	34·4	34·4

In a case like this it is obviously best to take the mean of series 2, 4, 5, and 6 as the true average, and to reject 1 and 3. The following table gives the comparison of Sirius, β Andromedæ, and Polaris with Capella. In the measurement of Sirius the times were less than 20^s, so that the limit of accuracy of the stop-watch, 0·2 seconds, introduced an error of more than 1 per cent. The magnitudes are calculated taking Capella as of magnitude 0·2.

Date	23.12.1918	23.12.1918	4.3.1919	4.3.1919
Star and spectral type	α Canis Majoris (A)	β Andromedæ (M)	α Aurigæ (G)	α Ursæ Minoris (F 8)
Volts per second × 10^4	588 610 626	11·48 11·36 11·30	148·2 148·2 148·2 149·1	24·93
Probable error of single observation × 10^4	12·7 = 2·1 %	0·061 = 0·54 %	0·3 = 0·2 %	
Mean value in volts per second × 10^4	608 ± 7·5	11·36 ± 0·036	148·4 ± 0·15	24·93
Height	22° 48′	39° 30′	51°	51°
Calculated magnitude at zenith	0·25 − 1·53 − 0·38 = − 1·67	0·25 + 2·79 − 0·13 = 2·91	(0·25 − 0·05 = 0·20)	0·25 + 1·94 − 0·05 = 2·14
Visual magnitude	− 1·6	2·4	(0·2)	2·1

It must be remembered that these magnitudes do not represent accurately either visual or photographic magnitudes, though they may be expected to approach the latter. β Andromedæ being of type M is therefore much fainter measured with the blue sensitive potassium cell than it would be measured visually. Similarly, Sirius being of type A appears somewhat brighter than it does visually.

Stellar Temperatures.

This leads to an important application of photoelectricity by the combination of a potassium cell and a cæsium cell alternately. The cæsium cell is predominantly sensitive in the yellow part of the spectrum, whereas the potassium cell is affected chiefly by the blue and violet. By using them alternately one obtains a sort of colour index, which should form a fairly accurate measure of the

temperature. Unless the cells could be calibrated with a known spectrum it would probably be advisable to calibrate the two cells with stars of known temperature, but it is easy to form a rough estimate of the sort of accuracy to be expected. If the wave-length at which the selective effect is predominant is 5500 Å. for Cs, as it is 4400 Å. for K, the ratio of the energies indicated by the two cells is

$$\left(\frac{\lambda_K}{\lambda_{Cs}}\right)^5 \frac{e^{\frac{1\cdot46}{\lambda_K T}} - 1}{e^{\frac{1\cdot46}{\lambda_{Cs} T}} - 1} = 0\cdot286\, \frac{e^{\frac{33200}{T}} - 1}{e^{\frac{26600}{T}} - 1}$$

so that the difference in magnitude is given by

$$2\cdot5 \log \frac{e^{\frac{33200}{T}} - 1}{e^{\frac{26600}{T}} - 1} - 1\cdot36.$$

When T is small this reduces to $T = \frac{7160}{\Delta m + 1\cdot36}$, Δm being the difference in magnitude measured with a cæsium cell and with a potassium cell. The type of curve is shown in fig. 4, from which

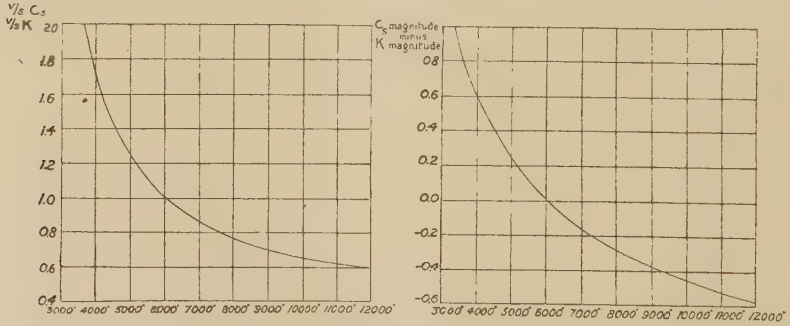


FIG. 4.

it appears that considerable accuracy may be hoped for. An error of 1 per cent., for instance, only leads to an error of about 100° in the temperature.

An instrument for this purpose has been designed and is in course of construction, but no results are as yet available.

Star Counts.

Another possible use of the photoelectric photometer is as a labour-saving device when making star counts in regions in which the average magnitude can be estimated, though whether this can be done sufficiently accurately appears doubtful.

Luminosity and Temperature of Nebulæ.

The most important application appears to be the measurement of the light received from extended surfaces. Thus, for instance, there does not seem to be any reliable evidence about the light obtained from nebulæ, though such information would be extremely valuable from many points of view. If one knew the equivalent stellar magnitude of a spiral nebula, for instance, it is clear that it is easy to deduce the number of luminous stars when the parallax is known from the observation of novæ. This will allow the nebulæ to be compared to the galaxy as regards stellar density. Owing to the weather, only a single measurement of the light received from the Andromeda nebula was carried out. The two readings were $3{\cdot}4.10^{-4}\ \frac{\text{volts}}{\text{sec.}}$ and $3{\cdot}32.10^{-4}\ \frac{\text{volts}}{\text{sec.}}$. Compared to β Andromedæ, whose magnitude is given as 2·4, this leads to an equivalent magnitude of 3·33 for the nebula. As mentioned above, β Andromedæ is of type M, and the potassium cell, therefore, indicates less than the visual magnitude. Suppose, however, magnitude 3·3, which is probably much too great, is taken as an illustration—for the weather was far too uncertain to allow one to attach very much weight to it—one finds that the number of stars of average brightness of the sun N is equal to $\frac{r^2}{12{\cdot}3}$, r being the distance measured in parsecs. Thus, for instance, if N is taken as 10^{10}, as is approximately the case in our system, $r = 350000$ parsecs, and the diameter, taken as 40′, comes to 4000 parsecs. It is clear that there is nothing inconsistent, therefore, in regarding this nebula as a galaxy very similar to our own.

A question that seems well worth studying by this means is the average temperatures of the spiral nebulæ. A point of view often adopted more or less consciously is that all the stars in the galaxy are approximately of the same age and are gradually radiating away the energy with which they were endowed at birth. If this is so for the galaxy, it is almost certainly true of spiral nebulæ. In this case either all the nebulæ must have been formed at the same time—a most improbable hypothesis—or one should find older and younger, *i.e.* colder and hotter, nebulæ. If the relative brightness of all nebulæ measured with the cæsium and the potassium cell does not vary very much, they are all more or less at the same temperature. If this is true, one is forced to the conclusion that the life of the galaxy is great compared to the life of a single star. This would imply the constant birth of stars, or the renewal of a star's life either by collision or some other means.

What has been said about the spiral nebulæ applies in large measure to the globular clusters. Since the magnitude of the giant stars in these is known, and these can be counted, the relative number of the other stars can probably be estimated.

The gaseous and the planetary nebulæ are perhaps rather more

difficult to treat directly photoelectrically than the spirals, since it is difficult to interpret the results of the photoelectric effect of a bright line spectrum. There is, however, little hope of progress in explaining the problems these objects present until some knowledge of the amount of light they emit has been obtained.

Photometry of Comets and Determination of their Mass.

Another important question to which a quantitative answer is lacking is the photometry of comets. There are two simple ways of obtaining an estimate of the mass of a comet (*M.N., R.A.S.*, 1911 December). It is probably generally admitted by now that a comet consists mainly of numerous particles. These will repel one another owing to radiation pressure unless they are greater than a certain size and the comet will scatter and disintegrate. This size is given by the equation

$$\left(\frac{4\pi}{3}R^3\rho\right)^2 f = \frac{4\pi R_0^2 \cdot \sigma T_0^4}{4\pi r^2} \cdot \frac{(\pi R^2)^2}{4\pi c}$$

or

$$R\rho = \frac{3}{8\sqrt{\pi}} \frac{R_0}{r} \sqrt{\frac{\sigma T_0^4}{fc}},$$

R being the radius, and ρ the density of the particle, and r its distance from the sun, whose radius is R_0 and temperature T_0. c is the velocity of light, f the constant of gravity, and σ Stephan's constant. The light received per cm.^{2} from a comet, L, at some distance r_0 from the sun and r_1 from the earth is $L_0 \frac{R_0^2}{r_0^2} \cdot \frac{N\pi R^2 A}{4\pi r_1^2}$, L_0 being the light received per cm.^{2} at distance R_0 from the sun. The mass of the comet $M = N \cdot \frac{4\pi}{3} R^3\rho$, or, inserting the above value for $R\rho$,

$$M = 2\sqrt{\pi}\sqrt{\frac{\sigma T_0^4}{fc}} \cdot \frac{r_0^2 r_1^2}{r R_0} \frac{L}{A L_0}.$$

All these quantities are known excepting the albedo A and the luminosity L, and this can readily be determined photoelectrically.

The second method gives an upper limit for M, or even an accurate determination if sufficient data are available. The radial force towards the sun is $\frac{M_0 Mf}{r_0^2} - \frac{4\pi r_1^2 L}{c}$. If the comet did not change at all, $r_1^2 L$ would be proportional to the inverse square of r_0. Since it does change, radiation pressure causes a deviation of the total force from the inverse square law. It is readily seen, therefore, that accurate photometry of L will enable one to determine this deviation from the inverse square law which, together with the deviation of the comet's orbit from the true ellipse, will allow one to evaluate M.

Photometry of Planets and Variability of the Sun.

There are several interesting problems connected with the accurate photometry of planets. Apart from the albedo, which may be determined accurately for different colours, the question as to whether the sun is a variable star or not could be cleared up once for all. This problem is difficult to treat directly, as any variation might easily be masked by changes in the atmospheric conditions. These changes are eliminated if the luminosity of planets relative to fixed stars is measured, due allowance of course being made for the obvious geometrical corrections. Any change of albedo would obviously only affect one planet at a time, whereas a change in the sun's luminosity would affect all the planets simultaneously.

The light from planets can of course be measured just as easily and accurately photometrically as the light from stars. The following table gives two typical measurements of Jupiter's luminosity. The whole duration of an experiment was less than ten seconds, and the time read with a stop-watch is of course confined to intervals of $0{\cdot}2^s$. This is probably the chief source of error.

JUPITER.

Date	23.12.1918			4.3.1919			
Volts per second $\times 10^4$	1384	1426	1412	1043	1043	1000	1000
					1000	1043	
Probable error of single observation $\times 10^4$	$14 = 1\%$			$14{\cdot}3 = 1{\cdot}4\%$			
Mean value in volts per second $\times 10^4$	1407 ± 8			$1021{\cdot}5 \pm 5{\cdot}9$			
Height	$61^\circ\ 24'$			52°			
Calculated magnitude at zenith	$-2{\cdot}18$ (Sirius $= -1{\cdot}6$)			$-1{\cdot}895$ (Capella $= 0{\cdot}2$)			
Magnitude given in *Nautical Almanac*	$-2{\cdot}2$			$-2{\cdot}0$			

Since Capella is of type G like the sun, the second measurement should be near the truth. If it is, it would indicate that the albedo of Jupiter is less for blue light than it is for white light, or that the planet shines partly with intrinsic light.

Photometry of Moon and Albedo of the Earth.

A further interesting application of photoelectric photometry enables one to determine the albedo of the earth. This may be of considerable importance for meteorology especially, since it should give a reasonably good average of the amount of cloud over a large fraction of a hemisphere.

If A_1 is the albedo of the earth and A_2 that of the moon, the

amount of light one would receive on the earth from a part of the moon not struck by the sun's rays is approximately

$$L_1 = L_0 \frac{\pi - \theta}{4\pi^2} \alpha^2 \frac{R_1^2}{r^2} A_1 A_2,$$

θ being the angle between sun and moon, R_1 the radius of the earth, r the distance of the moon, L_0 the light received from the sun per cm.^{2}, and α^2 the solid angle under observation.

Only one opportunity has offered up to date to measure the light from the dark part of the moon, and on this occasion, unfortunately, the iris diaphragm had not yet been calibrated, so that there is some uncertainty about α^2. The light was found to be equal to that from a star magnitude 0·42 on this occasion. Putting the magnitude of the sun as $-25{\cdot}93$, the Harvard photographic magnitude, one finds $\frac{L_0}{L_1} = 3{\cdot}47.10^{10}$. The best value of A_2 at present is probably 0·186, derived from the difference of visual magnitude of sun and full moon given by Harvard. At the time of measurement θ was approximately 4 hours or one radian, and α^2 may be taken as 300 square minutes, corresponding to an opening of nearly 20′. Inserting these values, one finds $A_1 = 0{\cdot}41$.

No claim to accuracy is made for this value, which is inserted more as an illustration than anything else. It has been found that the slightest haze distorts the measurement completely. Further, it is evident that A_2 should be determined in the same way as A_1, *i.e.* photoelectrically. The problem would, however, well repay careful investigation, using a potassium cell and a rubidium or cæsium cell.

Photometry of Solar Phenomena.

There are two other direct applications which need only be touched on briefly. The quantitative determination of the luminosity of various solar phenomena is urgently required before any theory can be checked, and this could often be carried out photoelectrically. The brightness of the corona, or the zodiacal light, for instance, is only given very vaguely, and even the protuberances and faculæ do not seem to have been measured with any great accuracy. The actual amount of light of various colours emitted by the umbra of a sun-spot again would be of considerable interest, and could easily be measured.

Photometry of Sky.

The second application is rather meteorological than astronomical, and is concerned with the amount of light scattered by the atmosphere and the opacity of the air. The first is readily found in daytime by measuring a given solid angle of sky at a definite distance from the sun. The second is given by the

absolute current produced by any star, provided it is not a variable. Auroræ may of course also be measured quantitatively, provided due precautions are taken to allow for the fact that the spectrum is not continuous.

Photometry of Photographic Plates.

Finally, it is worth while to call attention to the photoelectric photometry of the photographs of nebulæ, comets, and the like. Provided they are not over-exposed in any part—and this could be avoided if necessary by taking a series of plates with different exposures—there seems every hope that one could combine the two methods with advantage. The only difficulty appears to be the standard used for comparison. It seems doubtful whether a stellar image is to be recommended, as these are almost necessarily over-exposed locally. The best would be a standard wedge photographed with a known light on the edge of each plate. This should not present any difficulties, and would enhance the value of the plates considerably for later measurement. If it were desired to estimate temperatures, it would of course be necessary to take two separate photographs through different coloured screens.

Summary.

A detailed description is given of the method of preparing photoelectric cells for astronomical purposes. A full explanation of the electrometer employed in conjunction with them and of the method of mounting and use is offered.

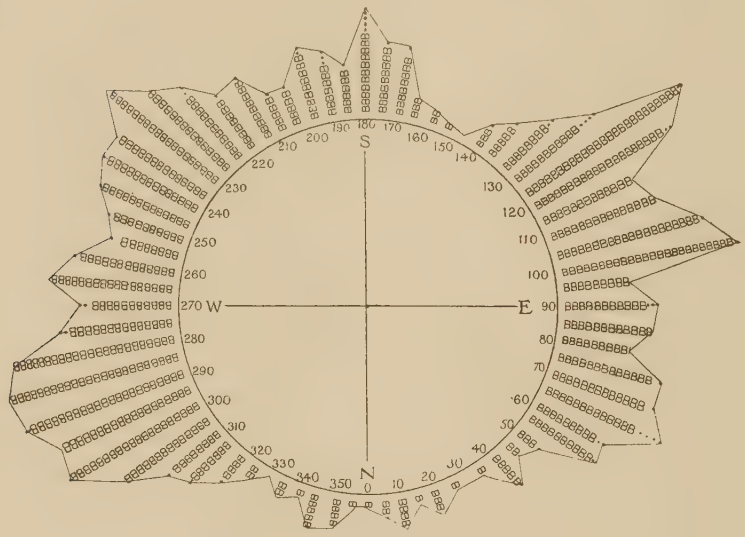
A number of applications are suggested, and where possible illustrated by preliminary measurements. The more important are the measurement of luminosity and temperature of stars and nebulæ, the luminosity of comets from which their mass may be derived, the brightness of planets by which any variability of the sun would become apparent, the determination of the earth's albedo by photometry of the dark part of the moon, and the measurement of the opacity of the air and luminosity of zodiacal light and auroræ.

Sidmouth,
1919 *March* 11.

Solar Prominences, 1918. By G. J. Newbegin.

In the year 1918 seventy-seven observations for solar prominences were made, one less than in 1917; 836 prominences were recorded, one more than in 1917, making a very equable comparison for the two years.

For International Solar Union requirements a deduction of 244 is necessary, leaving a remainder of 592 prominences with which to construct the accompanying diagram.



B = bright. . = faint.

Distribution of Prominences, 1918.

The diagram shows that the display of prominences has been greater in the South than in the North Polar region and generally; the mean daily average works out as 7·6.

The four hemispheres may be compared as under:—

North, 317	South, 398 = 715
East, 333	West, 382 = 715

south and west being in excess.

Dark absorption was recorded on twenty-seven occasions, 23 on bright and 4 on faint prominences.

Mean average height (I.S.U.), 46″·6.

Lyndale, Sutton, Surrey.

Diffraction Effects in Occultations by Planets.

By A. S. Eddington and J. W. Nicholson.

When an occultation of a star by a planet is observed, attention is usually directed to the question whether the disappearance is sudden, or whether there is a gradual dimming of the star which might be attributed to absorption in an atmosphere surrounding the planet. We shall not here examine the testimony on this point; some observers are confident that they have observed a gradual fading of the light, but records of these occultations are somewhat rare. Our object is to discuss whether, if the gradual disappearance is established, this should be taken as evidence of a planetary atmosphere. Owing to diffraction the disappearance in any case cannot be strictly instantaneous; it is therefore of interest to inquire whether diffraction is of sufficient magnitude to be appreciable in these observations, and in particular how it compares with the effect that is likely to be produced by a planetary atmosphere. It has been shown by one of us * that, in the case of occultations by the moon, diffraction would mask any effect due to the finite angular diameter of the star; but our general conclusion is that in planetary occultations it would be insufficient to account for the alleged gradual fading, and our result tends to support the popular interpretation.

The planet interposed between us and the star casts a shadow; and it is well known that a shadow (even for a point-source of light) has no abrupt boundary, but is fringed by diffraction bands. As the observer is borne into the shadow he passes through the diffraction bands, and the light from the star appears to vary. The variation of the light can be calculated from the mathematical theory of waves. Since our result is mainly negative, we omit details of the investigation, giving only the final formulæ.

Let a be the linear radius of the planet,

b its distance from the earth,

ρ the shortest distance of the geometrical ray from the centre of the planet,

so that $\frac{\rho - a}{b}$ is the angular distance of the star from the limb.

Introduce the quantity ψ given by

$$\psi = (\rho - a)\sqrt{\frac{2}{b\lambda}},$$

where λ is the wave-length of the light.

Further, let

$$C(\psi) = \int_0^{\psi} \cos\frac{\pi}{2}x^2 \,.\, dx \qquad S(\psi) = \int_0^{\pi} \sin\frac{\pi}{2}x^2 \,.\, dx.$$

Then the intensity, as ρ varies, is proportional to

$$\tfrac{1}{2}\{\tfrac{1}{2} + C(\psi)\}^2 + \tfrac{1}{2}\{\tfrac{1}{2} + S(\psi)\}^2.$$

* A. S. Eddington, *Monthly Notices*, 69, 178.

The formula still holds when the observer is inside the geometrical shadow, but in that case $\rho - a$, ψ, $C(\psi)$, and $S(\psi)$ are all negative. These results were obtained by treating the planet as a circular disc; but it will be seen that they are independent of the radius a, involving only $\rho - a$, the height of the ray above the surface. It is only the portion of the obstruction close to the geometrical ray that affects the intensity.

The following table gives the intensity of the star's light for different values of ψ, the normal brightness being taken as unity. In the fifth column the variation of light is converted into magnitudes. In the last column we give for illustration the corresponding height of the ray above the surface for Mars on 1918 April 11,* when its distance was 0·72 astronomical units; we have then $\sqrt{\frac{1}{2}b\lambda} = 160$ metres.

ψ.	$C(\psi)$.	$S(\psi)$.	Intensity.	Magnitude. m	Height above Surface. metres.
3·2	·466	·593	1·06	− ·06	500
2·88	·543	·406	0·95	+ ·06	450
2·56	·390	·578	0·98	+ ·02	400
2·24	·632	·494	1·12	− ·12	350
1·92	·413	·367	0·79	+ ·26	300
1·6	·365	·638	1·02	− ·02	250
1·28	·654	·674	1·345	− ·32	200
0·96	·774	·399	1·21	− ·21	150
0·64	·612	·135	0·82	+ ·22	100
0·32	·319	·018	0·47	+ ·82	50
0	0	0	0·25	+1·51	0
−0·32	− ·319	− ·018	0·132	+2·2	− 50
−0·64	− ·612	− ·135	0·073	+2·8	− 100
−0·96	− ·774	− ·399	0·043	+3·4	− 150
−1·28	− ·654	− ·674	0·027	+3·9	− 200
−1·6	− ·365	− ·638	0·019	+4·3	− 250

In this example the main diminution of the light begins when the ray passes 200 metres above the surface, and the star fades nearly two magnitudes by the time the ray grazes the surface. On the other hand, we might expect that atmospheric absorption would begin to be perceptible when the ray passes several thousand metres above the surface, and the fading from this cause would be perhaps twenty times more gradual.

On the date mentioned 1000 metres on Mars corresponded to 0″·002, and the planet moved through this distance in a quarter of a second. It would thus appear that in an ordinary occultation the atmospheric absorption might be just above, whilst the diffraction would be just below, the limit of perception. In the case of a close graze, extinction from both causes would be more gradual. Theoretically the diffraction effect might then be just observable; but the observer would have to be very fortunately situated.

* An occultation was observed in Australia on this date.

Photographic Magnitudes and Effective Wave-Lengths of Nova Aquilæ (1918) *determined at the Cambridge Observatory.*

(*Communicated by Professor A. S. Eddington.*)

The following photographic magnitudes of Nova Aquilæ were determined from measurements of the images of the nova and a comparison star obtained with equal exposures. For the first three nights α Aquilæ (adopted magnitude $1^{m}\cdot07$) was used as comparison star; from June 18 onwards β Aquilæ (adopted magnitude $4^{m}\cdot96$) was used. In both cases the nova and comparison star were near the same altitude, and no correction has been applied for difference of atmospheric absorption.

The photographs were taken with the Sheepshanks (Coudé) Equatorial. Usually the programme on each night consisted of two exposures on the nova, then four on the comparison star, then two more on the nova, all of 20^{s}, followed by a similar series of 40^{s} exposures. On the first four nights shorter exposures were given. The images of both stars were compared with a graduated series of images forming a scale, the scale being calibrated by comparison with photographs of the Pleiades, for which photographic magnitudes are known.

A parallel-strip grating was placed over the object-glass; usually the comparison was made between images of the same order in both stars, but sometimes it was more convenient to compare central images of β Aquilæ with first-order images of the nova, allowing for the theoretical difference of magnitude.

Photographic effective wave-lengths were determined differentially on each night by comparison with β Aquilæ, for which the wave-length 4326 (Lindblad's value for type K) has been adopted. A somewhat large correction to the measured wave-length, depending on the size of the images, was found and applied when required; but this effect probably renders the results unsatisfactory except when the nova and comparison star were near the same magnitude. For this reason we have rejected results before June 28: and of those printed the latest results are the most reliable. The exposures given turned out to be unsuited for the accuracy of determination of the wave-length, and the probable error is unduly large.

The evidence is insufficient to establish definitely a change of effective wave-length during the period of observation. From July 24 to August 20 the mean value is 4288, which, in a star of continuous spectrum, would correspond to type between G0 and G5. If the corresponding colour-index, $0^{m}\cdot7$, is applied to the photographic magnitudes given below, they are found to be fairly accordant with the visual determinations published by the Harvard and Radcliffe Observatories; but it is uncertain how far the ordinary relation of effective wave-length and colour-index holds for a nova.

The observations were made about 10^{h} G.M.T.

Magnitudes and Effective Wave-Lengths of Nova Aquilæ.

Date.	Photographic Magnitude.	Effective Wave-Length.	Date.	Photographic Magnitude.	Effective Wave-Length.
1918.	m		1918.	m	
June 13	1·20	...	July 13	4·43	4363
15	1·81	...	14	4·50	4302
16	1·99	...	18	4·34	4426
22	3·16	...	24	4·58	4285
23	3·38	...	25	4·61	4315
28	4·60	4271	27	4·68	4322
29	4·74	4279	29	4·50	4292
30	4·64	4395	Aug. 6	4·72	4287
July 2	3·89	4355	7	4·41	4313
3	3·50	4302	9	4·37	4328
9	4·18	4442	13	5·03	4221
11	4·33	4346	14	4·92	4293
12	4·61	4407	20	4·93	4222

The photographs were taken and the magnitudes measured by Mr. H. E. Green; the effective wave-lengths are from measures by Professor Eddington.

Further Observations of Nova Persei (1901).

By A. Stanley Williams.

The observations contained in the present paper (No. 6) are in continuation of those already communicated to the Society.* They were all made with a 6½-inch reflector, a power of 73 being always used, except where otherwise mentioned.

Table I., below, gives full particulars of the comparison stars made use of. Photometric measures of the magnitudes of four of these stars by Wendell at Harvard Observatory have been published in the *Astrophysical Journal*, **16**, 338, and are given in col. 3 of the table. Col. 5 contains the photometric magnitudes derived from Hagen's grades in col. 4 by means of Table II. in the first paper of this series, p. 339. Col. 6 gives the light-scale as determined from my own observations.

Table I.

Comparison Stars.

Star.	Hagen No.	Harvard Mag.	Hagen Grades.	Hagen Photo. Mag.	W. Steps.	W. Photo. Mag.
h	20	9·10	67	9·13	0	9·10
l	34	10·57	92	10·25	21	10·57
d	36	...	94	10·33	21	10·57
t	42	11·03	111	11·04	28	11·05
w	49	12·23	145	12·26	45	12·23
u	54	...	159	12·69	44	12·16
x	56	...	161	12·75	53	12·70

* *Monthly Notices*, **61**, 337, 396, 480, 550; **62**, 589.

When the step values according to col. 6 * were plotted on squared paper as abscissæ, and the Harvard magnitudes in col. 3 as ordinates, a straight line could be drawn through the dots indicating the results. From this it may be concluded that my light-scale is homogeneous within the limits 9 mag. to 13 mag., and that the magnitudes contained in the last column of Table II., below, are correct magnitudes on the Harvard photometric scale. The agreement with the Hagen magnitudes in col. 5 of Table I. is not so good.

TABLE II.

Observations.

Date.	G.M.T.	J.D.	Observations.	Mag. on Harvard Scale.
1902	h m	241+		
Aug. 24	15 30	5986	h10N12d, c10N	9·80
Sept. 3	14 35	5996	h5N15d, b5N	9·47
6	13 40	5999	h1·5N	9·20
12	14 00	6005	h5N14l, b3N	9·47
26	11 00	6019	h10N7l, b10N15d	9·96
Oct. 21	10 35	6044	h10N10l	9·83
Nov. 12	9 25	6066	h12N6l	10·08
Dec. 2	9 30	6086	h15N7l1t	10·10
18	9 00	6102	h15N6l	10·15
31	10 10	6115	h17N5l10t, N16t	10·17
1903				
Jan. 23	9 30	6138	h15N10l6t	9·98
28	9 40	6143	h20N5l7t	10·28
Feb. 17	7 30	6163	N5l, N15t	10·12
Aug. 3	14 30	6330	h18l5toN18u	10·80
Sept. 2	14 40	6360	l3N=t18u	11·01
15	13 00	6373	l4Not14u	10·97
25	12 44	6381	l6N2t10u	10·93
Oct. 16	10 50	6404	t7N12u	11·46
30	9 42	6418	l7t5N12u	11·51
Nov. 9	10 35	6428	l15t=N18u	11·01
1904				
Jan. 5	9 25	6485	t6N10u	11·47
9	6 20	6489	toN14u	11·08
Aug. 2	15 00	6695	l14t12N11w	11·67
Sept. 3	15 18	6727	t12N2w10x	12·06
Oct. 3	12 56	6757	N3w	12·02
28	9 48	6782	t9N17u	11·43
Dec. 13	9 20	6828	t6N17w, N20u	11·21
18	7 17	6833	t10N11w	11·61
30	7 20	6845	t8N13w	11·50

* The value of one step is 0·07 magnitude.

TABLE II.—*continued.*

Date.	G.M.T.	J.D.	Observations.	Mag. on Harvard Scale.
1905	h m	241+		
Feb. 28	8 47	6905	t12N5w	11·88
Aug. 5	14 54	7063	t16N5w	11·95
Sept. 20	16 18	7109	t14N8w	11·80
Oct. 19	10 32	7138	t9N11w	11·58
21	14 46	7140	t10N9w	11·67
29	10 57	7148	t7N10w	11·60
Dec. 19	9 56	7199	t8N10w	11·57
1906				
Jan. 13	10 30	7224	t8N10w	11·57
Aug. 19	13 25	7442	t3N14w	11·26
19	15 38	7442	t7N10w	11·53
21	14 06	7444	t10N8w	11·68
27	13 34	7450	t14N1w	12·15
28	14 39	7451	t14N0w, N9x	12·16
Sept. 11	12 00	7465	t12N1w	12·09
19	11 15	7473	t13N4w	11·95
25	11 13	7479	t13N3w	12·01
Oct. 10	10 12	7494	t14N5w	11·92
23	11 08	7507	t10N4w	11·89
28	12 32	7512	t10N6w	11·79
Nov. 11	8 05	7526	t12N4w	11·89
15	9 05	7530	t10N5w	11·84
Dec. 7	8 34	7552	t10N5w	11·84
21	11 11	7566	t14N5w	11·92
1907				
Jan. 11	11 03	7587	t10N5w	11·84
17	9 23	7593	t10N5w	11·84
Aug. 19	13 42	7807	t12N2w	12·06
Sept. 7	14 28	7826	t10N4w	11·89
Oct. 4	10 37	7853	t8N5w	11·78
Dec. 5	10 36	7915	t7N5w	11·74
1909				
Aug. 10	14 23	8529	t12N, w1N	12·09
Sept. 6	14 41	8556	t9N3w	11·93
14	13 24	8564	t8N0w	12·08
Oct. 21	10 22	8601	t9N1w	12·04
Nov. 5	10 57	8616	t10N0w	12·11
1911				
Sept. 22	15 35	9302	t12N3w, N11x	12·10
Oct. 17	14 10	9327	t15N, w2N, N5x	12·32
Dec. 11	10 18	9382	t12N, N3w	11·99

The following are a few notes relating to the colour and appearance of the nova, and other matters:—1902 September 3, with power 225 the colour of N was white, with perhaps a slight bluish tinge. September 6, N appeared bluish and not so sharply defined as h. September 26, N slightly bluish. October 21, the colour of h was white, that of N bluish; the latter hazy-looking, quite different from the bright white disc of h. 1903 September 2, N seemed bluish. 1904 September 3, cloud directly after. 1905 September 20, power 110. 1909 September 14, perhaps some cloud about.

The nova was looked for in the autumn of 1918, but it must have been near the limit of visibility with the 6½-inch reflector, and probably not much above 13 mag. No attempt was made to identify it among several faint stars in its vicinity.

At their first appearance novæ, when bright, attract almost universal attention, so much so that the wealth of observational material becomes almost embarrassing from its very richness. But in their fainter stages these stars have been unduly neglected. Most of them appear to be still liable to more or less large fluctuations of brightness. The general impression seems to be that such fluctuations are irregular or spasmodic in their nature, and without signs of periodicity. The question whether the changes are periodic or irregular is not an unimportant one, since the existence of periodicity, however rough or imperfect, would suggest that the cause of the variations is to be sought for in the stars themselves, and not to some outside cause, such as the passage of a star through nebulosity of variable density.

Now the observations of Nova Persei, when faint, do seem to be suggestive of periodicity, though perhaps rough or imperfect. Table III. col. 1 gives the dates, extracted from the foregoing observations, when the star was relatively bright; col. 2, the intervals in days between two of the dates in col. 1; col. 3, the number of periods of 55 days, assumed as the period of variation; and col. 4, the corresponding interval in days. At the end of the table have been added three observations when the star was relatively bright, made by E. E. Barnard, and published in *Popular Astronomy*, 25, 556 (1917).

TABLE III.

Nova Bright.	Interval. Days.	No. of Periods.	No. of Days.
J.D. 2415999			
6138	139	3	165
6381	243	4	220
6428	47	1	55
6489	61	1	55
6828	339	6	330
7442	614	11	605
2420868	384	7	385
1252	219	4	220
1471			

It should be remembered that the dates in col. 1 are those when the star was merely relatively bright, and not necessarily those when it was at or even near a maximum of brightness. Also that the regularity of an actual periodicity might well be masked by other internal causes operating in the star itself, so that it would seem possible occasionally for a maximum apparently to fail altogether. The material available is insufficient for certainty, and all that can be said is that there is a suggestion of a periodical variation in about 55 days.* It is well known that in the early stages of its decline there were distinct indications of variation in a period of a few days. Has the length of the period increased since then to about 55 days? One of about 48 days, or less, would better satisfy the earlier results in Table III.

St. Mawes:
1919 *March* 1.

Note on the Absorption Spectrum of Nova Geminorum II., 1912 *March.* By F. J. M. Stratton, D.S.O., M.A.

(*Communicated by the Director, Solar Physics Observatory, Cambridge.*)

The close resemblance of the absorption spectrum of Nova Geminorum II. on 1912 March 15 to that of α Cygni has been pointed out by Professor Newall and the writer in a previous note (*M.N., R.A.S.*, 73, 380). The publication of further investigations on this Nova was delayed for the sake of completing the measurement and reduction of a number of spectrograms most kindly lent in 1913 by Dr. F. Schlesinger of Allegheny Observatory and by Dr. Küstner of Bonn Observatory to fill up gaps in the Cambridge series of spectrograms. The measurements and reductions were nearly completed in 1914, when the war compelled the temporary postponement of further investigations. In the interval another brilliant Nova has appeared, and it is in the hope that the following note may be useful to those who are working at Nova Aquilæ III. that it is published as a preliminary note on the spectrum of Nova Geminorum II.

The absorption lines measured on the Cambridge, Bonn, and Allegheny plates from March 15 onwards are for the first fortnight accounted for very largely as α Cygni lines displaced to the violet by the same multiple of their wave-lengths as is given by the displacements of the hydrogen absorption lines. The displacement varies from day to day, the general condition being a steady *diminution* in displacement from one day to the next. Accompanying and superposed on this decreasing displacement there comes at irregular time intervals a fresh outburst of absorption lines with a much greater displacement, which diminishes in its turn. At first the new lines appear for hydrogen alone, but

* Or, perhaps, in some sub-multiple of this period.

enhanced metallic lines (of iron and titanium) and later helium lines are soon also represented by absorptions displaced to the violet by the same multiple of the wave-length as in the case of the hydrogen lines. Interpreting the phenomena in terms of the Doppler effect, we are led to regard the observed displacements as indicating a succession of pulses. Each pulse starts with a large initial velocity which slowly dies down ; the absorption corresponding to each pulse also grows weaker, and is ultimately lost through falling back into the accompanying strong radiation band. Below is given for March the series of factors κ which give the displacement of the absorption lines to the violet by the formula $\delta\lambda = \kappa\lambda$.

TABLE I.

Values of $10^4\kappa$ *for* 1912 *March.*

Date.	Pulse I.	Pulse II.	Pulse III.	Pulse IV.	
Mar. 15	19				
16	15	46			
17	10	44			
18	5				
19	5	32	56		
20		30	56		
21		28	54		
22		28	50		
23					
24					
25		27	48		
26		26	47		
27		24			
28		25		53	
29		27		54	Two plates taken on the same evening. IV. for the second plate is very uncertain.
		28		59 ?	
30		32		54	
31		33		59	

Up to March 27 the bulk of the absorption lines in the spectra are explicable in terms of an α Cygni spectrum (or possibly two) sharing in the displacements of the hydrogen lines. But on March 28 this key fails to explain the spectrum, and at the same time another point is noticed for the first time: the hydrogen absorptions can apparently only be interpreted as moving to the violet instead of to the red. This is continued on March 29 and 30, and on the latter date both of the hydrogen absorptions are doubled by the emergence of a bright central radiation. The general absorption spectrum becomes very complex, but it can be explained as follows :—Taking the centres of the two absorption doubles to represent two pulses, we find survivals of an α Cygni

spectrum displaced as if travelling with Pulse II., while the most prominent features of the spectrum are now (March 30) best summarised as a γ Orionis spectrum displaced with Pulse IV. Oxygen, helium, and nitrogen are the most prominent elements present, in addition to hydrogen.

The evidence for this is shown in Table II., which gives a list of wave-lengths of absorption lines measured on plates taken at Allegheny and Cambridge on 1912 March 30, computed sources, identifications, and comparisons with the laboratory wave-lengths. The factors for the displacements are deduced from the absorptions for Hβ, Hγ, Hδ on the same date:

	Nova λ		$10^4\kappa$.	
Pulse II.	4846·1	Hβ 4861·5	32	Mean κ II. 0·0032
	4327·1	Hγ 4340·6	31	
	4089·0	Hδ 4102·0	32	
Pulse IV.	4835·0	Hβ 4861·5	55	Mean κ IV. 0·0054
	4317·2	Hγ 4340·6	54	
	4080·4	Hδ 4102·0	53	

The lists of stellar lines have been taken from the following publications:—for α Cygni, table iii. of *Catalogue of Four Hundred and Seventy of the Brighter Stars*; for γ Orionis, *Comparison of the Spectra of Rigelian, Crucian, and Alnitamian Stars* (Publications of the Solar Physics Committee).

TABLE II.

Absorption Lines in Nova Geminorum II., 1912 *March* 30.

Nova λ.	Displaced with Pulse II.	α Cygni.	Displaced with Pulse IV.	γ Orionis.	Residual, Nova – comparison.
3902·3	3914·8	3913·6 pTi	3924·4	3926·7 He	+1·2 II. −2·3 IV.
3914·8			3935·9	3933·8 pCa	+2·1
3921·7	3934·2	3933·8 pCa			+0·4
3933·9					Undisplaced K
3951·2			3972·5	3968·6 pCa 3970·2 H	
3957·9	3970·4	3968·6 pCa 3970·2 H			
3968·6					Undisplaced H
3973·6			3995·1	3995·3 N	−0·2
3988·2			4009·7	4009·4 He	+0·3
4003·6			4025·2	4026·3 He	−1·1
4019·8			4041·6	4041·5 N	+0·1
4051·4			4073·3	4070·0 O 4072·4 O 4076·1 O	Hazy absorption, 10 tm. broad

TABLE II.—*continued.*

Nova λ.	Displaced with Pulse II.	α Cygni.	Displaced with Pulse IV.	γ Orionis.	Residual, Nova – comparison.
4097·5			4119·6	4119·5 O	+0·1
4100·5			4122·6	4121·0 He	+1·6
4120·7			4142·9	4143·9 He	−1·0
4145·2			4167·6	4169·1 He	−1·5
4148·5	4161·8	4163·8 pTi			−2·0
4159·0	4172·3	4173·6 pFe 4173·7 pTi			
4210·4			4233·1	4233·3 pFe	−0·2
4219·7	4233·2	4233·3 pFe	4242·5	4241·9 N	−0·1 II. +0·6 IV.
4231·7			4254·6	4253·8 O	+0·8
4244·7	4258·3	4258·6	4267·6	4267·3 pC	−0·3 II. +0·3 IV.
4283·2	4296·9	4296·6 pFe			+0·3
4291·7	4305·4	4303·3 pFe			+2·1
4294·4	4308·2	4308·1 pTi	4317·6	4317·3 O 4319·8 O	
4300·0	4313·8	4313·0 pTi 4315·1 pTi			
4305·4			4328·6	4327·6 O	+1·0
4355·6			4379·1	4379·7 N	−0·6
4365·1			4388·6	4388·1 He	+0·5
4372·3	4386·3	4385·5 pFe	4395·9	4396·1 O	+0·8 II −0·2 IV.
4380·4	4394·4	4395·2 pTi			−0·8
4383·5	4397·5	4399·9 pTi			−2·4
4391·1			4414·8	4415·1 O 4417·1 O	
4401·6	4415·7	4417·0 pFe			−1·3
4407·7			4431·5	4432·9 N	−1·4
4411·8			4435·6	4437·7 He	−2·1
4421·8			4445·7	4447·2 N	−1·5
4429·3	4443·5	4444·0 pTi			−0·5
4446·6			4470·6	4471·6 He	−1·0
4456·9	4471·2	4471·6 He	4481·0	4481·4 pMg	−0·4 II. −0·4 IV.
4473·6	4487·9	4489·3 pFe 4489·5 pTi			
4487·7	4502·1	4501·4 pTi			+0·7
4505·7	4520·1	4520·4 pFe	4530·0	4530·1 N	−0·3 II. −0·1 IV.
4521·6	4536·1	4534·1 pTi			+2·0
4528·3			4552·8	4552·6 N	+0·2

TABLE II.—*continued.*

Nova λ.	Displaced with Pulse II.	α Cygni.	Displaced with Pulse IV.	γ Orionis.	Residual, Nova – comparison.
4536·4	4550·9	4549·6 pFe 4549·8 pTi			
4566·1			4590·8	4591·1 O	−0·3
4568·7	4583·3	4584·0 pFe			−0·7
4572·0			4596·7	4596·3 O	+0·4
4576·5			4601·2	4601·7 N	−0·5
4582·8			4607·5	4607·3 N	+0·2
4588·5			4613·3	4614·0 N	−0·7
4596·4			4621·2	4621·6 N	−0·4
4605·2			4630·1	4630·7 N	−0·6
4613·3			4638·2	4639·0 O	−0·8
4615·6	4630·4	4629·5 pFe	4640·5	4641·9 O	+0·9 II. −1·4 IV.
4617·6			4642·5	4643·3 N	−0·8
4624·7			4649·7	4649·3 O	+0·4
4637·1			4662·1	4661·8 O	+0·3
4651·4			4676·5	4676·3 O	+0·2
4685·9			4711·2	4713·2 He	−2·0
4894·9			4921·3	4922·1 He	−0·8
4909·5	4925·2	4924·1 pFe			+1·1
4987·7			5014·6	5015·7 He	−1·1
5003·5	5019·5	5018·6 pFe			+0·9

A glance at the signs of the last four residuals will show that the enhanced lines of iron λ 4924, 5018 attributed to Pulse II. are both present, along with the helium lines λ 4922, 5016 attributed to Pulse IV.

Only five of the γ Orionis lines of strength greater than 2 are missing in the Nova spectrum: these are 3964·9 He, 4035·1 N, 4153·7 O and S, 4567·9 Si (III.), and 4574·8 Si (III.); of these 4567·9 Si (III.) may be represented by a weak line measured on one plate only at 4543·0. Sixty-six lines in the Nova have been accounted for, leaving fifteen residuary lines, mostly weak ones. The strongest outstanding lines are the following:—

Nova.	Source in Pulse II.	Source in Pulse IV.	Possible Origin.
4152·9	4166·2	4175·3	4176·2 N, γ Orionis.
4270·2	4283·9	4293·2	4292·2 4295·0 } γ Orionis.
4497·1	4511·5	4521·4	
4580·1	4594·8	4604·8	
4665·9	4680·8	4691·1	
4868·2	4883·8	4894·5	
4914·8	4930·5	4941·4	4941·2 O 4943·1 O
4920·3	4936·0	4946·9	

On the whole, the superposition of the two spectra—α Cygni in Pulse II. and γ Orionis in Pulse IV.—explains the absorption spectrum very satisfactorily. There are naturally a considerable number of blends from the superposition of lines from the two pulses, but the crucial evidence for the new identifications is given by the complete correspondence with the well-known group of lines of oxygen and nitrogen between $\lambda\lambda$ 4600 and 4650.

Solar Physics Observatory,
Cambridge:
1919 *March* 11.

On the Classification of Long-period Variable Stars. Third Paper: The Influence of the Period on the Shape of the Curves in Phillips's two Classes. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In examining various possibilities connected with the classification of variables by their light-curves, I was led, by a path which need not be here followed, to arrange the stars in order of period. The result was striking, when due note was taken of Phillips's classification (*B.A.A. Journal*, 1916 October 25). I was led, however, to make a (comparatively slight) modification by adopting the sign of $\cos\phi_2$ as an absolute criterion. This meant transferring from Group I. to Group II.

	S Cygni	R Leonis	R Bootis
for which	$\phi_2 = 233^\circ$	114°	126°

while on the other hand

S Lyræ	and	R Persei
$\phi_2 = 67^\circ$		75°

are transferred from Group II. to Group I. The 6 stars omitted by Phillips are assigned without difficulty: in Group II.

χ Cygni	V Cygni	Y Virginis	T Aquarii
$\phi_2 = 131^\circ$	138°	154°	171°

while we have in Group I.

RV Herculis	and	T Hydræ
$\phi_2 = 51^\circ$		$\phi_2 = 10^\circ$

2. We will begin with the more extensive Group II., which is rearranged as in Table I.

TABLE I.

Phillips's Group II., modified so that cos ϕ_2 is always negative, and arranged according to period.

Name.	Harv. No.	Gal. Lat.	Period.	Freq.	ϕ_2.	ϕ_3.	k_2'.	k_3'.
		°	d		°	°		
S Cassiop.	011272	+ 9	630	...	135	107	8	4
V Delphin.	204318	−16	529	...	106	63	24	6
Z Cassiop.	233956	− 5	492	3	97	65	19	11
RU Herculis	160625	+46	485	4	197	187	17	8
R Cassiop.	235350	−12	430	5	118	89	21	12
R Hydræ	*132422*	+38	425	13	165	181	16	4
R Cygni	193449	+13	424	29	103	93	20	4
U Cygni	201647	+ 6	418	23	138	334	9	3
Y Cassiop.	235855	− 7	410	28	149	95	31	10
SX Cygni	201130	− 4	409	48	152	89	8	12
R Androm.	001838	−24	406	67	97	66	20	5
χ Cygni	194632	+ 3	405	50	131	154	11	7
U Herculis	162119	+41	403	26	154	111	24	7
W Androm.	021143 *a*	−15	395	17	182	122	13	4
R Aquarii	*233815*	−71	387	16	176	160	44	17
S Serpentis	151714	+53	377	28	176	125	28	7
S Virginis	*132706*	+53	377	42	159	161	15	5
R Pegasi	230110	−45	373	53	164	133	28	23
R Leo. Min.	093934	+49	371	67	141	106	15	6
R Gemin.	070122 *a*	+14	370	71	98	35	18	7
U Orion.	054920	− 2	368	53	126	106	18	8
S Tauri	042309	−25	365	50	149	105	45	18
R Cancri	081112	+24	362	83	169	154	16	10
R Comæ	115919	+78	362	48	122	33	23	9
S Coronæ	151731	+57	362	17	104	69	24	10
R Piscium	012502	−60	344	13	154	113	30	13
T Virginis	*120905*	−55	339	17	177	159	31	9
o Ceti	*021403*	+64	332	23	118	59	24	9
S Cygni	200357	+13	327	31	233	206	14	8
R Tauri	042209	−25	325	42	180	107	21	6
S Ceti	001909	−52	321	53	178	195	13	3
R Aquilæ	190108	0	320	77	134	89	9	8
R Herculis	160118	+44	318	125	136	102	12	8

TABLE I.—*continued.*

Name.	Harv. No.	Gal. Lat.	Period.	Freq.	ϕ_2.	ϕ_3.	k_2'.	k_3'.
			d					
R Corvi	*121418*	+43°	318	100	162°	147°	26	9
S Pegasi	231508	−48	317	36	105	93	19	10
R Leonis	094211	+44	312	21	114	154	12	4
R Ophiu.	*170215*	+13	302	18	192	195	4	6
R Urs. Maj.	103769	+44	301	15	126	75	18	6
R Delphin.	201008	+14	284	16	166	171	15	5
T Androm.	001726	−36	281	32	110	80	9	5
W Herculis	163137	+41	280	48	148	125	13	5
S Aquarii	*225120*	−65	278	23	103	21	21	6
R Sagittarii	*191019*	−15	269	14	147	94	4	6
T Urs. Maj.	123160	+57	258	14	94	356	13	4
V Virginis	*132202*	+57	250	15	179	182	8	2
R Draco.	163266	+38	247	12	122	60	7	1
R Bootis	143227	+66	225	13	126	202	2	2
R Scorpii	*161122 a*	+19	224	18	136	137	24	4
Y Virginis	*122803*	+58	219	11	154	28	18	6
T Aquarii	*204405*	−30	203	...	171	242	5	2
S Scorpii	*161122 b*	+19	177	.	167	140	11	1

3. The first two columns require no comment. In the third column is given the galactic latitude, and it is seen that a physical fact formerly suspected in a different form now emerges from its disguise:

The stars of long period affect the Milky Way (in both groups).

Taking the mean latitude (without regard to sign) for consecutive groups of 5 stars, we get the following series, the last group containing 6 stars:—

TABLE II.

Period.	Mean. Gal. Lat.	Period.	Mean Gal. Lat.
513	18°	333	43°
417	14	318	37
399	31	296	30
374	43	267	47
364	37	224	38

The first star with a really high galactic latitude is R Aquarii, fifteenth on the list. The mean latitude for the first 14 stars is only 17°, that for the remaining 37 stars is 40°.

4. In the fourth column of Table I. is given the period as recorded by Phillips; it will be seen in what follows that a slight change in the period might sensibly affect the accordance of some of the results noticed. But it will be therefore the more necessary to avoid bias in assigning alterations of period.

5. In the fifth column is given the "frequency" of period, assigned as follows:—The interval between consecutive periods might be taken as half that between the preceding and following periods. But it will be seen that there are 3 stars of period of 362 days, which would give zero interval in the absence of fractions of a day. To reduce such accidental errors, let us associate also the interval between the pair of stars next outside these, *i.e.* the next-but-one stars. This gives four times the average interval in the neighbourhood. To give more weight to the closer determination, multiply it by 2 and add, getting eight times the average interval. Thus in the case quoted, R Comæ, we have 365 and 344 for the outside pair and 362 and 362 for the inside: total

$$21 + 2 \times 0 = 21,$$

so that the mean interval is $21/8 = 2{\cdot}6$ days. But since the whole measure is provisional and arbitrary, and since we are going to take the reciprocal, so as to give frequency rather than interval, it is needless to divide by the 8. The reciprocal of 21 is 48, as given in the fifth column. The reason for this inversion is that it is in many ways more convenient to have, near the ends of the table, small numbers such as 3 and 4 tending to zero rather than large ones tending to infinity: but there are, nevertheless, some advantages in retaining the intervals rather than the frequencies. For the present it is sufficient to notice the general character of the variation in frequency. We note maximum and minimum values simply by inspection, as in the first columns of Table III.:—

Table III.

Max.	Period.	Add.	Sum.	Min.	Period.	Add.	Sum.
67	406	0	406	16	387	0	387
83	362	45	407	13	344	45	389
125	318	90	408	15	301	90	391
48	280	135	415	12	247	135	382
		Mean	409				387

6. There is a clear variation in a cycle of about 45 days. The range of variation is larger in the middle of the series, and if we lose sight of it near the ends, the disappearance is quite possibly real. There is a trace of an additional minimum at 418 days, and though we cannot assign its position very accurately, its

magnitude cannot be far from 23. Using these five minima and the four maxima, we can assign the range as in Table IV.

TABLE IV.

Ranges of Frequency and Interval.

Period.	Frequency.			Interval.		
	Max.	Min.	Range.	Max.	Min.	Range.
(429)		23		43		
406	67		47		15	38
387		16	59	63		50
362	83		68		12	58
344		13	91	77		67
318	125		111		8	64
301		15	72	67		52
280	48		35		21	54
247		12		83		

For comparison with the use of frequencies, they have been reinverted to form intervals. The rise in range is clearly shown in the middle of both series.

7. Various attempts have been made to obtain a more definitive discussion, but the general conclusion is that the material in its present state scarcely justifies such attempts. The main facts are pretty clear: a variation in about 45 days (which is so convenient a number for use, being an exact submultiple of 360°, that it has been adopted as it stands) with a zero-point at about 330 days; and a range which is greatest at this point, diminishing outwards from it.

8. We now come to ϕ_2 in the sixth column. The values vary a good deal, and we must do something to reduce the accidental errors. In view of the possibility just noticed of a 45-day cycle, we must not group together at present periods differing by more than about 10 days, but we may form such groups for ϕ_2 and for other elements at the same time.

[TABLE.

TABLE V.

Groups of Stars in Table I.

Period.	No. of Stars.	ϕ_2.	ϕ_3.	k_2'.	k_3'.
488	2	147	126	18	10
426	3	129	121	19	7
412	3	146	53	16	8
405	3	127	110	18	6
391	2	179	141	29	10
374	4	160	131	22	10
368	3	124	82	27	11
362	3	132	85	21	10
342	2	166	136	30	11
328	3	177	124	20	8
319	4	152	133	15	7
314	2	110	124	16	7
302	2	159	135	11	6
281	4	132	99	14	5
264	2	120	45	8	5
248	2	151	121	8	2
223	3	139	122	15	4

9. The variation in ϕ_3 is greater than that in ϕ_2, and the eye is at once caught by the small values for ϕ_3 at periods 412, 368, and 264. With these as a guide, we can see that both ϕ_2 and ϕ_3 respond to the 45-day cycle as follows:—

TABLE VI.

Maxima and Minima of ϕ_2 and ϕ_3.

Period.	Epoch.	ϕ_2.		ϕ_3.		Ranges.	
		Max.	Min.	Max.	Min.	ϕ_2.	ϕ_3.
		°	°	°	°	°	°
412	412		146		53		
391	413·5	179		141		44	73
368	413		124		82	48	56
342	409·5	166		136		39	33
314	404		110		124	53	12
302	414·5	159		135		41	37
(272)	407		(126)		(72)	29	56
248	405·5	151		121		19	24
223	403		139		122		
Mean	409	156	129	133	91	39	46

10. In one case only (shown in brackets) has the simple mean of two adjacent groups of Table V. been taken: otherwise the numbers are simply transferred from Table V. The first column gives the actual value of the period in Table V., without any attempt at interpolation. In the second column these are reduced to a common epoch by successive additions of a half cycle (22·5 days). Considering the mode of formation of the numbers, the accordance is fairly good; there is some suggestion of a progressive diminution, indicating that the adopted period is too short, whereas the suggestion of Table III. is rather the other way. But the accordance of the mean epochs of Table III. and Table VI. at the value 409 days is satisfactory. A maximum in both ϕ_2 and ϕ_3 corresponds, apparently, to a minimum of "frequency" or a maximum of "interval."

11. The ranges in the last two columns are deduced by the obvious process of comparing a maximum with the mean of two adjacent minima, or *vice versa*. Neither series shows any pronounced tendency to be greater in the middle, that for ϕ_3 showing a distinctly contrary tendency. But it seems doubtful whether we can attach significance to the differences as yet. Here again several attempts at more rigid discussion only suggested that as yet the material scarcely justifies it.

12. Returning to Table IV., the quantities k_2' and k_3' (which are simply Phillips's coefficients for the second and third harmonics, divided by that of the first) show no variation in 45 days. But they clearly show progressive changes. Since they are not affected by the 45-day cycle, we can take means for larger groups, say every 10 stars of Table I. There are 51 stars in all, and we omit the first with period very much longer than anything else. The means for the remaining 50 are as below:—

TABLE VII.

Mean Values of Coefficients of 2nd and 3rd Harmonics (in terms of that for the first) for Groups of ten Stars; from Table I.

Period.	k_2'.	k_3'.	Calc.	O−C.	ϕ_2.	ϕ_3.
443	·185	·075	·074	−·001	132°	90°
383	·214	·091	·086	+·005	151	121
344	·241	·095	·096	−·001	158	120
303	·137	·066	·055	+·011	139	123
235	·113	·034	·045	−·011	140	110
Mean	·177	·072				

13. The relationship $k_3' = 0{\cdot}4k_2'$ gives the values of k_3' deduced from k_2' in the column Calc., and seems to represent the observations well.

14. For the same groups of 10, the mean values of ϕ_2 and ϕ_3 were formed. They are both greater in the middle, and com-

parison of the mean of the two extreme groups with the mean of the other three gives the relationship

$$\phi_3 = 1{\cdot}5\phi_2 - 104^\circ,$$

similar to that found by Phillips.

15. We now turn to Group I. and arrange it similarly in order of period. The values of ϕ_2 are in the neighbourhood of 360°, and 360° is added in cases requiring it for continuity.

TABLE VIII.

Phillips's Group I., modified so that cos ϕ_2 is always positive, and arranged according to period.

Name.	Harv. No.	Gal. Lat.	Period.	Freq.	$\phi_{2'}$.	ϕ_3.	$k_{2'}$.	$k_{3'}$.
		°	d					
S Cephei	213678	+19	486	...	283	204	7	5
U Cygni	201647	+ 6	461	...	388	188	17	6
R Aurigæ	050953	+ 8	449	11·9	302	216	17	10
T Cassiop.	001755	− 8	443	17·5	324	191	28	11
S Lyræ	190925	+ 6	438	14·3	427	299	25	5
T Draco.	175458	+29	426	9·3	445	183	15	2
S Orionis	*052404*	−21	413	7·2	316	168	18	10
T Cephei	210868	+14	386	7·4	295	205	10	7
R Lyncis	065355	+24	379	7·3	402	211	6	6
R Serpent.	154615	+46	359	6·4	443	248	15	5
S Can. Min.	072708	+13	330	7·2	393	207	12	4
R Can. Ven.	134440	+72	328	8·1	374	205	15	5
S Herculis	164715	+33	303	7·2	283	187	7	7
T Hydræ	*085008*	+23	289	7·7	370	360	9	8
R Camelop.	142584	+33	269	11·1	367	213	11	4
S Bootis	141954	+58	266	18·9	358	258	6	2
V Androm.	004435	−27	259	18·5	442	169	10	6
S Hydræ	084803	+28	256	11·0	336	189	4	4
U Ceti	*022813*	−61	235	8·7	415	241	15	3
S Urs. Maj.	123961	+55	223	10·1	367	263	16	6
R Persei	032335	−17	210	14·9	435	336	4	4
U Virginis	124606	−68	207	19·6	345	197	13	7
RV Herculis	165631	+35	200	18·5	411	72	17	4
S Libræ	*151520*	+30	192	14·7	397	186	7	4
R Arietis	021024	−34	186	11·8	373	197	14	7
R Ceti	*022000*	−55	167	11·2	440	226	15	5
T Herculis	180531	+22	165	10·8	411	160	6	2
R Virginis	123307	+69	145	...	370	216	8	5
R Vulpec.	205923	−16	137	...	403	153	8	4

16. The galactic latitudes are again small at the head of the table, the first really high latitude being for R Can. Ven., period 328 days, and the twelfth in order. The mean for the first 11 stars is 18°, and for the remaining 18 is 42°. We may take the means for groups of 5 stars for all the elements, to study slow changes: the last group has only 4 stars.

TABLE IX.

Gradual Changes in Group I. (means of 5).

Period.	Gal. Lat.	ϕ_2.	ϕ_3.	k_2'.	k_3'.
d	°	°	°		
455	9	345	220	19	7
393	27	380	203	13	6
304	35	357	234	11	6
248	46	384	224	10	4
199	37	392	198	11	5
154	40	406	189	9	4
Mean	32	377	211	12·2	5·3

There is thus a steady fall in both k_2' and k_3' (which are again related as 10 to 4, but have only two-thirds of their values in Group II.), and the fluctuations in ϕ_2 and ϕ_3 go opposite ways. The formula

$$\phi_3 = 438^\circ - 0{\cdot}6\phi_2$$

represents them fairly well, the coefficient of ϕ_2 being sensibly larger than that given by Phillips. But we must be prepared for short-period fluctuations in ϕ_2 and ϕ_3, which may modify the above figures.

17. The frequencies are sensibly less than for Group II., and have been given to another place, distinguished by a decimal, so as to keep the unit the same as for Group II. The large value for period 207 and the small for 235 suggest a cycle of 60 days, as indicated below:—

TABLE X.

Fluctuation of 60 Days in the Frequency of Group I.

Period.	Epoch.	Frequency.		Range.	Interval.		Range.
443	443	17·5			57		
413	443		7·2	5·2		139	43
386	446	7·4		0·6	135		13
359	449		6·4	1·3		156	26
328	448	8·1		1·1	124		20
(296)	446		(7·5)	6·0		(133)	44
266	446	18·9		10·8	53		71
235	445		8·7	10·6		115	63
207	447	19·6		9·5	51		50
(176)	446		(11·5)			(87)	
Mean	446	14·3	8·3	5·6	84	126	41

18. Forming short groups now to reduce accidental errors, we have—

TABLE XI.

Short Groups from Table VIII.

Period.	No. of Stars.	ϕ_2.	ϕ_3.	k_2'.	k_3'.
455	2	345	202	17	8
441	2	376	245	26	8
420	2	380	176	16	6
383	2	348	208	8	6
359	1	443	248	15	5
329	2	384	206	14	4
296	2	326	274	8	8
268	2	362	236	8	3
258	2	389	179	7	5
229	2	391	252	16	4
208	2	390	266	8	6
193	3	394	152	13	5
166	2	426	193	10	4
141	2	386	184	8	4

19. We now look for hills and valleys separated by intervals of 30 days if we take the frequency as a guide. The high peak in ϕ_2 at period 359 depends on an isolated star: a better guide is the valley at period 296. Using this as a starting-point, we form the following table:—

TABLE XII.

Period.	Epoch.	ϕ_2.			ϕ_3.		
		Max.	Min.	Range.	Max.	Min.	Range.
d	d	°	°	°	°	°	°
450	450	360			224		
420	450		380	−26		176	+40
383	443	348		−64	208		− 4
359	449		443	−77		248	−41
329	449	384		0	206		−56
296	446		326	+46		274	−53
268	448	362		+ 4	236		− 9
242	452		390	−14		216	+35
208	448	390		−10	266		+71
180	450		410	−22		173	+52
141	441	386			184		
Mean	448	372	392	−23	226	217	+ 4

20. The existence of a periodicity of 60 days is thus very doubtful, the coefficient being reversed in both ϕ_2 and ϕ_3. There is a systematic appearance about the reversal, which might, for instance, be an "interference" effect, due to the coexistence of two terms with mean value 60 days and repeating in about 7 cycles,

say 56 days and 64 days. But our present material scarcely warrants the elaboration of such a hypothesis. It is safer to base the dependence of the elements on the period from the evidence of the slower changes than on that of the fluctuations.

In reading the proof it was noticed that a cycle of 70 days seems to suit both ϕ_2 and ϕ_3 as well as the frequency (less well).

Summary.

1. Phillips's classification, based on relations between ϕ_2 and ϕ_3, only needs slight modification when we adopt as characteristic of Group I. a positive sign for $\cos \phi_2$, and for Group II. a negative sign.

2. The characters of both groups then show relation to the period of variation.

3. For Group II. the phases ϕ_2 and ϕ_3 show fluctuations as the period changes by multiples of 45 days: and similar fluctuations appear in the "frequency" with which the stars occur at any particular period (§§ 5–11).

4. The coefficients of the harmonics in Group II. do not show these fluctuations, but change steadily with the period. The phases ϕ_2 and ϕ_3 also show gradual changes (§ 12).

5. The "frequency" for Group I. shows fluctuations in 60 days, but the representation of these in ϕ_2 and ϕ_3 is doubtful (§§ 17–20).

6. But all four characters (ϕ_2, ϕ_3, k_2', and k_3') of Group I. show gradual changes with the period (§ 16).

7. The stars of long period in both groups affect low galactic latitudes (§ 3 and § 16).

[*Note added March* 22.—Among the particulars tabulated by Phillips, the ranges (of magnitude between max. and min.) and the brightnesses (indicated by the sum of twelve ordinates of the light-curve) were omitted from the above tables, since they did not seem to affect the main argument, so far as could be seen at the time of writing. But a point of some interest has since presented itself concerning the ranges, which seem to indicate a possible junction between the two groups. If we take means for periods by 50 days, as below,

Period.	Group I.		Group II.		II.–I.
	No. Stars.	Mean. Range.	No. Stars.	Mean. Range.	
d					
Over 470	1	2·8	4	5·7	+2·9
421–470	6	3·8	3	5·5	+1·7
371–420	3	4·4	11	5·1	+0·7
321–370	3	4·9	12	5·1	+0·2
271–320	2	4·6	11	5·1	+0·5
221–270	5	4·5	6	4·8	+0·3
171–220	5	4·2	3	4·0	−0·2
Below 170	4	4·5	0		

Does one group pass into the other at the junction of the short periods, and, if so, which is the direction of transition?

In discussing the Baxendell stars, Miss Blagg often made several analyses of light-curves of the same star for different periods. Looking through the list and arranging in order of period, we have—

67·5 days—S Vulpec; no analysis.
69·5 days—R Scuti; Group I.
70·5 days—R Sagittæ; Group I., though the whole curve shifts 180°, retaining its shape.
147 days—S Aquilæ (*M.N.*, 77, 576). Early observations (1863–1869) would give an erratic Group II. ($\phi_2 = 230°$); the later ones (1877–1887) give Group I. ($\phi_2 = 14°$).
188 days—R Arietis, Group I. Progressive change in ϕ_2 from 344° to 30°.
210 days—R Persei. Earlier observations (1861–1866 and 1866–1869) give Group I. ($\phi_2 = 354°$ and 62°). Later observations (1878–1882 and 1884–1887) give Group II. ($\phi_2 = 108°$ and 122°). The progressive change is noteworthy.
224 days—R Bootis, II. to I. and back to II.? $\phi_2 = 142°$, 46°, 226°.
278 days—S Delphini, II. to I. and back to II.? $\phi_2 = 204°$, 26°, 198°, 184°.
313 days—R Leonis; doubtful in Baxendell. $\phi_2 = 90°$, 116°, 54°, 90°. Phillips makes it II.

These are the only cases of possible change, and they suggest some instability in the shorter periods: but more evidence is needed.]

Note on Mr. Pocock's Paper "On a Certain Relation between the Elements of Planets and Satellites." By H. H. Turner.

In December last a paper was communicated to the Society under circumstances described by me in an introductory note. My attention has since been called to the fact that the whole argument of the paper is substantially contained in a letter by Mr. Selig Brodetsky, printed in *Nature* for 1914 March 12 (93, 33). It will be noticed that Mr. Pocock refers in his first paragraph to *Nature*, 92, 724, where attention was drawn to M. F. Ollive's paper. This note in *Nature* apparently led both Mr. Pocock and Mr. Brodetsky to the same reply; and it seems clear that the former saw the printed reply of the latter after he had written his own note, but before he had posted it. He thereupon laid it aside, which explains why it was found among his papers as already mentioned.

Preliminary Values of Variation of Latitude at Greenwich during 1918.

(*Communicated by the Astronomer Royal.*)

In continuation of the results published in previous years, preliminary values of the latitude variation at Greenwich during 1918 are here given. The results are based upon the measurements of 213 plates taken with the Cookson floating zenith telescope. Unfortunately these are not evenly distributed throughout the year, and unfavourable observing conditions during the last two months of the year gave a small weight to the determination during that period. Advantage was taken of a fine spell in February of this year to secure as many further plates as possible, and the measures of an additional 18 plates have been utilised to determine the course of the latitude variation curve.

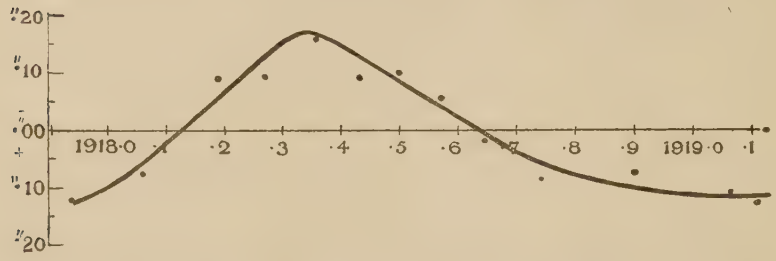
The method of reduction of the measures was generally similar to that explained in Mr. Jones's paper (*M.N.*, **75**, 42), but revised star-places and proper-motions have been determined and utilised, the group corrections have been redetermined, and temperature corrections have been applied. These have been deduced from the observations made with the instrument during the past seven years, which are now being discussed for the determination of the aberration constant.

The quantities under the heading "Variation" are observed *minus* mean latitude. The weight is the number of plates upon which each value is based.

Latitude Variation at Greenwich.

Date.	Variation.	Weight.
	″	
1917·94	+ ·124	18
1918·06	+ ·068	20·5
·19	− ·087	24·5
·27	− ·092	17·5
·36	− ·166	33·5
·43	− ·093	21
·50	− ·098	18
·57	− ·055	16
·65	+ ·016	26
·74	+ ·085	21·5
·90	+ ·074	13·5
1919·06	+ ·114	10·5
·11	+ ·121	7

A table of corrections applicable to observed N.P.D.'s for variation of latitude on selected dates is also included. These



were read from a smooth curve drawn through the observed values plotted to scale on a diagram.

Corrections applicable to observed N.P.D.'s. (*Longitude of Greenwich.*)

1918 Jan. 1	−·10	1918 May 1	+·17	1918 Sept. 1	−·02
11	−·08$_5$	11	+·17	11	−·03
21	−·06$_5$	21	+·16	21	−·05
Feb. 1	−·04	June 1	+·14	Oct. 1	−·06
11	−·02	11	+·12$_5$	11	−·07
21	+·01	21	+·10$_5$	21	−·08
Mar. 1	+·02$_5$	July 1	+·08$_5$	Nov. 1	−·08$_5$
11	+·05	11	+·07	11	−·09
21	+·08	21	+·05$_5$	21	−·10
Apr. 1	+·10$_5$	Aug. 1	+·04	Dec. 1	−·10
11	+·12$_5$	11	+·02	11	−·10$_5$
21	+·15	21	·00	21	−·11
				31	−·11

Decentred Lens-Systems. By A. E. Conrady.

In general discussions of the properties of lens-systems as well as in the computation of the data of new designs it is always assumed that the surfaces are accurately centred, that is, that all the centres of curvature lie on an exact straight line known as the optical axis of the system.

In the making of the real lenses this centring depends entirely on the care and skill of the workers, and can only be attained within limits which, assuming good workmanship, certainly are narrow, but which must nevertheless be regarded as of finite magnitude.

It follows that no actual lens-system possesses an optical axis in the rigorous sense of the theoretical discussions and calculations; in reality, the centres of curvature are located within small distances

of the intended axis, and are scattered around this in various azimuths according to chance.

The results of this departure from the assumed conditions do not seem to have been systematically investigated hitherto. Opticians know from practical experience that in an otherwise well-corrected system any decided centring error causes uniform coma over the entire field, accompanied in gross cases by one-sided colour-fringes, and they look out for this defect; it is therefore rarely seen by users of lenses. The fact, however, that the discovery of this defect in the preliminary workshop-tests of carefully made systems is not a very rare or unusual event proves that the residual centring errors are not negligible quantities.

Nothing appears to be known as to possible additional effects which might slightly distort the image without sensibly affecting its definition, which would therefore escape detection by the maker and user, and might thus introduce systematic errors into those extremely refined measurements of large fields which play so important a part in modern photographic methods of research.

The new method of dealing with the aberrations of oblique pencils, which I gave in my paper for the meeting of 1918 November, lends itself to a simple solution of this problem, and proves that such distortional effects *do* result from errors of centring.

In fig. 1 let OF represent the optical axis of an ideally centred optical system, AP one of the refracting surfaces with centre of curvature at C, OO_1 the object as presented to it by the surfaces in front of it, *i.e.* OO_1 is the image of the real object produced by the intervening refracting surfaces. Then my method of discussing the aberrations begins with an ideal image, free from aberration, which would be formed by thin pencils along the radial lines OC and O_1C, and would yield an image at F and F^1 according to the simple laws of pencils at very small angles of incidence. Now suppose that the refracting surface is tilted so that its centre now falls at C′. We can again trace radial pencils through the new position of the centre which will yield an image $F'F_1'$ tilted at an angle of the same order of magnitude as that by which the refracting surface is tilted, and which will be distorted according to the ordinary laws of central projection because we are now taking an *oblique* view of the surface in which the object OO_1 lies. This well-known distortion is that seen in snapshots of tall buildings taken on a plate inclined from the vertical, and is there characterised by convergence of vertical lines and by a falsification of the relative distances of horizontal lines, but in such a way that all straight lines in the objects are still rendered as straight lines in the image.

For the very small tilts which we are now considering the mathematical law of this distortion is a very simple one. The position angles of extra-axial points with reference to the centre of the field are not affected, but their distance d from it is altered by $b \cdot d^2 \cdot \cos a$ if a is the position-angle reckoned from the direction defined by CC′ and b a small constant. It is easily shown that any number of small tilts in different azimuths of successive

refractive surfaces lead to a final resultant distortion of precisely the same analytical form. This part of the effect of centring errors therefore becomes inextricably mixed with the same type of effect resulting from imperfect "squaring on" of the lens-system and of the plate-holder (or eyepiece), and is of no separate interest.

The real image is, however, produced by pencils of considerable aperture, which in the case of the more remote parts of a large field may not even include the thin radial pencil. These pencils of finite aperture are then subject to aberrations, and we must next examine the effect of centring errors on these aberrations.

Proceeding as in the November paper, we take an end view of the refracting surface along the radial line F_1A_1 in fig. 1 and obtain fig. 2, in which the large circle represents the lens-surface, the small circle the diameter of the oblique pencil passing through it, and A_1 the point where the radial line F_1C pierces it. Then the ray passing through any point R on the circumference of the pencil will suffer a lateral spherical aberration with reference to F_1 equal to $k(A_1R)^3$ if k is the aberration-constant for that surface, and the resulting displacement will be in the direction of A_1R. So far we have assumed that the surface is perfectly centred. Now consider the decentred surface. The line O_1F_1' (fig. 1) will pierce the surface in some point A_1', which we will define by the distance $A_1A_1' = r$ and by the position-angle $PA_1A_1' = O$. The pencil of rays, on the other hand, will cut the tilted surface on a line which differs only by negligible small quantities of the third order from the original position. The ray through R will now suffer lateral spherical aberration $= k(A_1'R)^3$, and this will lie in the direction of $A_1'R$.

Drawing a parallel to A_1P through A_1' and adopting the nomenclature of the former paper, viz. $A_1Q = Y$, $QR = y$, angle $RQR' = E$, we can take from the diagram

$$(A_1'R)^2 = (Y + y\cos E - r\cos O)^2 + (y\sin E - r\sin E)^2.$$

As before, we shall have the components of the aberrational displacement

$$\text{Horizontal component} = k(A_1'R)^3\,\frac{Y + y\cos E - r\cos O}{A_1'R}$$

$$\text{Vertical component} = k(A_1'R)^3\,\frac{y\sin E - r\sin O}{A_1'R}$$

and introducing the value of $(A_1'R)^2$, collecting terms and making slight reductions, we obtain the components of the lateral aberration:

$$\text{Horiz. comp.} = k(y^3\cos E + Yy^2(2 + \cos 2E) + 3Y^2y\cos E + Y^3) - k\,.\,r(y^2(2\cos O + \cos(2E - O)) + 2Yy(3\cos E\cos O + \sin E\sin O) + 3Y^2\cos O) + \text{terms in } kr^2 \text{ and in } kr^3$$

$$\text{Vert. comp.} = k(y^3\sin E + Yy^2\sin 2E + Y^2y\sin E) - k\,.\,r(y^2(2\sin O + \sin(2E - O)) + 2Yy(\sin E\cos O + \cos E\sin O) + Y^2\sin O) + \text{terms in } kr^2 \text{ and in } kr^3.$$

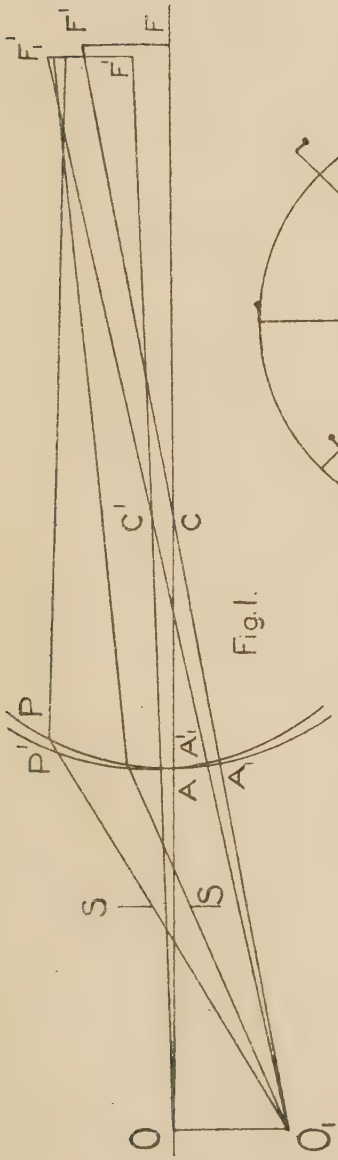


Fig. 1.

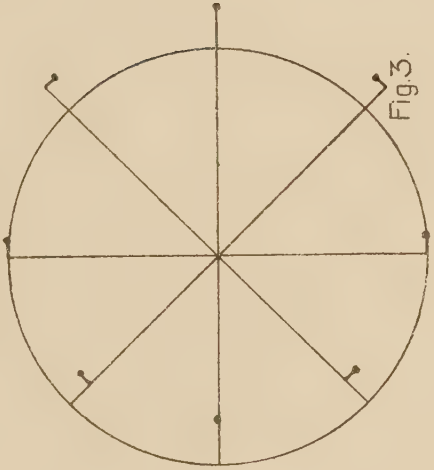


Fig. 3.

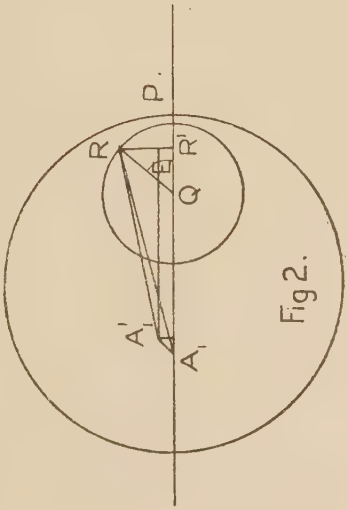


Fig 2.

The first line in each component represents the Seidel-aberrations of the perfectly centred surface, which thus reappear. They were discussed in the former paper, and do not interest us in the present investigation. The important terms resulting from the centring error appear in the second lines, and will be discussed. There are additional terms involving the second and third power of the centring error; optically these are quite interesting, but for our present purpose they may be safely ignored as utterly evanescent.

In most cases the centring defect will be chiefly due to one surface, for as its magnitude is proportional to the aberration-constant, a sensible effect will usually only be produced when it happens that a surface for which the constant has a high value is untrue. In ordinary object-glasses the contact-surfaces answer to this description.

The investigation may, however, easily be extended to any number of slightly decentred surfaces. As in the former paper, the displacements arising at any surface may be transferred to the following or to the final image by applying the magnification. We can also express the y and Y of every surface in terms of the constant semi-aperture S of the stop and of the angle V of the field of view. We can thus collect the algebraical sum of all the terms in the final image-surface. By applying simple transformations familiar to astronomers these sums again assume the analytical form of those found for a single surface, but with more or less *independent* coefficients for the terms in S^2, in SV, and in V^2. For a system of any number of decentred surfaces we thus obtain the final result, that the aberrational displacement of any ray due to the centring errors is of the form

$$\begin{aligned}\text{Horiz. comp.} &= p_1 S^2(2\cos\phi + \cos(2E-\phi)) \\ &\quad + p_2 SV(3\cos\psi\cos E + \sin\psi\sin E) + 3p_3 V^2\cos\chi \\ \text{Vert. comp.} &= p_1 S^2(2\sin\phi + \sin(2E-\phi)) \\ &\quad + p_2 SV(\cos\psi\sin E + \sin\psi\cos E) + p_3 V^2\sin\chi,\end{aligned}$$

in which p_1, p_2, and p_3 are constants determining the magnitude of the resultant centring defects, and ϕ, ψ, and χ angles indicating their orientation in the field in the same sense in which the angle O indicated the direction of the decentration in our investigation of the effect at a single surface.

We will now discuss the three terms separately:—

1. The terms in S^2. As they stand, these define the displacement of any ray from its ideal position at the final focus of the thin radial pencils in cartesian co-ordinates, the "horizontal" component being taken along a radial line of the field and the "vertical" component in the tangential direction. If we refer the co-ordinates to a new axis forming the angle ϕ with the original one, they become

$$\begin{aligned}x &= p_1 S^2(2+\cos(2E-\phi)) \\ y &= p_1 S^2\sin(2E-\phi),\end{aligned}$$

and on comparison with the Seidel-aberrations are immediately recognised as an orthodox coma-figure in the direction defined by the angle ϕ. For any given semi-aperture S this is constant all over the field (for it is independent of V), and evidently represents the uniform coma which was mentioned in the opening paragraphs as the only known effect of centring errors. As it is of the same amount and direction throughout the field of view, it does not affect the relative distances or position angles of points in it, and therefore is harmless as far as systematic errors in measurements are concerned. Like the Seidel coma, it grows with the square of the aperture.

2. The terms in VS. These assume their simplest analytical form by turning the co-ordinates through an angle equal to $\frac{1}{2}\psi$. The co-ordinates then become

$$x = p_2 \mathrm{SV} \frac{\sin \frac{3}{2}\psi}{\sin \frac{1}{2}\psi} \cos (\mathrm{E} - \tfrac{1}{2}\psi)$$

$$y = p_2 \mathrm{SV} \frac{\cos \frac{3}{2}\psi}{\cos \frac{1}{2}\psi} \sin (\mathrm{E} - \tfrac{1}{2}\psi).$$

The analogy with the Seidel-astigmatism is unmistakable, and closer discussion (which is really very interesting) shows that the terms represent astigmatic ellipses varying in length, eccentricity, and orientation in different parts of the field of view. But as this is a perfectly symmetrical deformation of the image points it is quite harmless, for it will of course be borne in mind that in any respectable instrument all these terms are bound to be very small.

3. There remain the terms V^2, namely,

$$x = 3p_3 \mathrm{V}^2 \cos \chi$$

$$y = p_3 \mathrm{V}^2 \sin \chi.$$

These are the only ones of astronomical interest, for they represent an extremely unpleasant dislocation of the image. As they contain neither S nor E they affect all the rays equally, just like the Seidel-distortion. Taken by themselves, the terms therefore represent simply a displacement of the sharp image.

The angle χ marks a fixed direction in the field. If we consider a point at any given distance from the centre of the field in the direction of χ, we shall obtain a radial displacement of the image $= 3p_3 \mathrm{V}^2$, and in the opposite direction a displacement $= -3p_3 \mathrm{V}^2$. For points at right angles to the direction of χ we shall have no radial displacement, but a tangential one $= p_3 \mathrm{V}^2$ always in the direction of χ. In any other direction both components have finite values, with the result that we obtain displacements in both the radial and tangential direction as roughly indicated in fig. 3. Straight lines in any position or direction other than that of the *diameter* of the field in the direction of χ are rendered as curved lines in the image.

If sensible, this effect of centring errors will therefore be

extremely objectionable. It must be borne in mind that the absence of the uniform coma-effect does *not* necessarily imply that this distortion is also absent. I have tried to estimate its reasonably possible magnitude, and feel confident that it could not reach one second of arc in the marginal parts of the ordinary astrographic plate. But in such instruments as the one used for the Franklin-Adams charts it might easily go into whole seconds.

Fortunately it will be a comparatively simple matter to test any given photographic instrument for the presence of this error. Let a rich field of stars near the meridian be photographed with instrument say West of the pier. Let the instrument be reversed to East of the pier and the plate turned through 180° in its own plane. If the same field is exposed again, it will be possible to secure the second impression everywhere very close to the first. There will thus be only a small parallel displacement plus a small difference of position angle between the first and second images of each star, and it should not be difficult to allow for these and to ascertain the direction and magnitude of the centring error. As an alternative two different plates could be exposed in the two positions of the instrument and a glass-positive from one of the negatives superposed upon the other negative, on Evershed's principle. In the absence of the error, perfect superposition would be attainable; in its presence, the reduction of the discrepancies to the possible minimum would everywhere display twice the aberrational centring error.

This seems such an obvious test that very possibly it has already been carried out with a view to the discovery of unsymmetrical displacements in the field, but in any case the knowledge of the law of distribution of the displacements caused by residual errors of centring should prove helpful in deducing the best possible corrective terms from a test of this description.

Imperial College of Science and Technology:
1919 *Jan.* 9.

Postscript.—My attention has recently been called to a paper by Mr. S. D. Chalmers, M.A. (*Proc. Phys. Soc.*, 30, 100, 1918), in which the lateral aberration was used for the purpose of deducing the third-order aberrations of lens-systems in a manner very similar to that employed in my paper of 1918 November (*M.N.*, 79, 60) on the same subject. So far as this method of attacking the problem is concerned, Mr. Chalmers can therefore undoubtedly claim priority.

1919 *March* 15.

Numerical Differences for the year 1923 *between Professor E. W. Brown's Tabular Places of the Moon and the Places according to Hansen's Tables* (1857) *with Newcomb's Corrections* (1878).

(*Communicated by P. H. Cowell, D.Sc., F.R.S.*)

The following table exhibits for every noon and midnight of the year 1923 the excess in longitude, latitude, and parallax of the places of the moon according to the new tables of Professor E. W. Brown, F.R.S., over the places according to Hansen's tables with Newcomb's corrections.

JANUARY.

	Longitude.		Latitude.		Parallax.	
	Noon.	Mid.	Noon.	Mid.	Noon.	Mid.
	″	″	″	″		
1	+5·7	+5·6	+1·5	+1·3	+·46	+·45
2	5·6	5·5	1·3	1·1	·46	·45
3	5·5	5·4	1·1	1·0	·45	·46
4	5·2	5·2	1·0	0·8	·47	·45
5	5·1	5·0	0·8	0·7	·45	·46
6	4·8	5·0	0·5	0·5	·48	·47
7	4·9	4·9	0·5	0·5	·48	·47
8	4·9	4·8	0·5	0·4	·49	·48
9	4·8	4·8	0·5	0·5	·47	·49
10	4·8	4·6	0·5	0·5	·47	·47
11	4·3	4·5	0·6	0·5	·46	·45
12	4·4	4·2	0·5	0·6	·47	·47
13	4·3	4·3	0·5	0·6	·48	·46
14	4·0	4·0	0·6	0·6	·46	·46
15	3·8	3·7	0·6	0·7	·48	·47
16	3·7	3·6	0·6	0·6	·47	·47
17	3·7	3·6	0·7	0·7	·47	·48
18	3·7	3·8	0·8	0·7	·48	·47
19	3·8	3·9	1·0	0·9	·47	·47
20	4·1	4·3	0·9	1·0	·47	·46
21	4·3	4·6	1·0	1·0	·48	·46
22	4·7	5·0	1·2	1·2	·46	·47
23	5·3	5·5	1·2	1·3	·46	·45
24	5·8	6·1	1·2	1·2	·47	·47
25	6·1	6·2	1·4	1·3	·46	·46
26	6·6	6·8	1·2	1·1	·47	·46
27	6·9	7·2	1·2	1·2	·48	·48
28	7·5	7·5	1·1	1·1	·49	·47
29	7·5	7·5	1·1	1·1	·48	·46
30	7·5	7·5	1·1	1·0	·47	·46
31	7·1	7·0	1·0	1·0	·45	·47

FEBRUARY.

	Longitude.		Latitude.		Parallax.	
	Noon.	Mid.	Noon.	Mid.	Noon.	Mid.
	″	″	″	″		
1	+6·8	+6·7	+1·1	+1·0	+·46	+·46
2	6·4	6·0	1·1	1·1	·47	·46
3	5·7	5·4	1·0	1·0	·47	·47
4	5·1	4·8	1·0	0·8	·48	·48
5	4·6	4·5	1·0	1·0	·47	·47
6	4·3	4·2	1·0	1·0	·47	·46
7	4·1	3·9	0·9	1·0	·46	·47
8	3·7	3·6	1·0	1·1	·45	·46
9	3·5	3·4	1·0	0·8	·45	·46
10	3·3	3·4	0·9	0·7	·46	·47
11	3·3	3·3	0·7	0·8	·47	·48
12	3·3	3·4	0·7	0·7	·47	·46
13	3·2	3·1	0·7	0·8	·47	·48
14	3·0	3·1	0·8	0·8	·48	·49
15	3·2	3·2	0·9	0·9	·48	·46
16	3·3	3·6	1·0	1·0	·47	·46
17	3·7	3·9	1·0	1·0	·47	·47
18	4·0	4·1	1·1	1·0	·46	·46
19	4·3	4·5	1·0	0·9	·46	·46
20	4·7	5·0	1·0	1·0	·46	·47
21	5·3	5·7	1·0	1·0	·46	·47
22	6·0	6·4	0·9	0·9	·46	·48
23	6·6	6·8	0·9	0·9	·48	·47
24	6·9	7·0	0·7	0·7	·47	·48
25	7·3	7·5	0·6	0·6	·49	·49
26	7·7	8·0	0·5	0·5	·48	·48
27	8·1	8·2	0·5	0·3	·48	·48
28	8·1	8·0	0·4	0·4	·46	·47

MARCH.

	Longitude.		Latitude.		Parallax.	
	Noon.	Mid.	Noon.	Mid.	Noon.	Mid.
	″	″	″	″		
1	+7·7	+7·5	+0·3	+0·3	+·46	+·47
2	7·1	6·8	0·4	0·5	·47	·48
3	6·4	6·0	0·6	0·8	·49	·49
4	5·5	5·0	0·9	1·1	·49	·48
5	4·5	4·1	1·2	1·2	·48	·47
6	3·9	3·4	1·4	1·5	·47	·48
7	3·2	2·9	1·5	1·5	·46	·46
8	2·9	2·6	1·5	1·5	·47	·45
9	2·5	2·5	1·7	1·6	·47	·45
10	2·3	2·3	1·6	1·5	·46	·47
11	2·3	2·3	1·5	1·4	·46	·45
12	2·5	2·7	1·3	1·3	·46	·46
13	2·8	2·9	1·3	1·3	·46	·46
14	2·9	2·8	1·3	1·3	·48	·47
15	2·9	3·0	1·3	1·3	·47	·47
16	3·1	3·2	1·3	1·2	·47	·48
17	3·5	3·8	1·2	1·2	·48	·46
18	4·2	4·3	1·1	1·1	·45	·45
19	4·5	4·7	1·0	1·0	·44	·45
20	4·9	5·1	1·0	1·0	·45	·45
21	5·3	5·7	1·0	0·9	·48	·48
22	6·1	6·6	0·9	0·8	·48	·48
23	6·9	7·2	0·8	0·7	·47	·48
24	7·5	7·6	0·6	0·6	·47	·46
25	7·7	7·8	0·6	0·5	·48	·48
26	7·9	8·1	0·4	0·3	·48	·47
27	8·2	8·4	0·3	0·3	·48	·48
28	8·6	8·7	0·3	+0·2	·48	·47
29	8·5	8·2	0·0	0·0	·47	·45
30	7·8	7·3	0·0	0·0	·48	·47
31	7·0	6·4	0·0	+0·1	·48	·49

APRIL.

	Longitude.		Latitude.		Parallax.	
	Noon.	Mid.	Noon.	Mid.	Noon.	Mid.
	″	″	″	″		
1	+6·0	+5·7	+0·2	+0·5	+·49	+·50
2	5·3	4·9	0·6	0·7	·49	·49
3	4·4	3·9	0·9	1·0	·49	·46
4	3·4	2·9	1·2	1·3	·47	·47
5	2·5	2·3	1·4	1·6	·47	·47
6	2·2	2·1	1·6	1·7	·47	·47
7	1·9	1·9	1·8	1·9	·46	·46
8	1·8	1·8	1·9	1·9	·47	·46
9	1·9	2·0	1·8	1·9	·46	·47
10	2·2	2·4	1·9	1·9	·46	·45
11	2·6	2·8	1·9	2·0	·46	·45
12	3·0	3·3	2·0	2·0	·46	·47
13	3·5	3·7	2·0	1·9	·47	·47
14	3·9	4·2	1·8	1·7	·48	·47
15	4·5	4·9	1·6	1·5	·47	·46
16	5·2	5·4	1·4	1·4	·46	·45
17	5·7	5·9	1·4	1·2	·45	·44
18	6·3	6·4	1·2	1·1	·44	·43
19	6·8	7·1	1·0	0·9	·46	·45
20	7·5	7·7	0·9	0·7	·48	·45
21	8·2	8·5	0·7	0·6	·47	·46
22	8·8	9·0	0·5	0·5	·45	·45
23	9·1	9·2	0·3	0·3	·44	·45
24	9·3	9·3	0·2	0·2	·46	·45
25	9·3	9·4	0·1	0·1	·47	·46
26	9·4	9·3	0·2	0·2	·47	·47
27	9·0	8·4	0·2	0·3	·48	·48
28	7·9	7·4	0·4	0·4	·48	·49
29	6·8	6·3	0·5	0·4	·49	·50
30	6·0	5·6	0·5	0·6	·51	·50

MAY.

	Longitude.		Latitude.		Parallax.	
	Noon.	Mid.	Noon.	Mid.	Noon.	Mid.
	″	″	″	″		
1	+5·0	+4·6	+0·5	+0·7	+·50	+·49
2	4·3	3·6	0·8	0·9	·50	·49
3	3·1	2·7	1·2	1·3	·49	·48
4	2·3	2·9	1·6	1·5	·49	·49
5	1·9	1·7	1·7	1·8	·51	·49
6	1·7	1·7	1·7	1·8	·47	·47
7	1·7	1·7	1·8	1·9	·48	·47
8	1·7	1·5	1·9	1·9	·46	·47
9	1·8	1·9	1·9	2·1	·47	·46
10	2·2	2·4	2·1	2·2	·46	·47
11	2·7	2·9	2·2	2·1	·46	·47
12	3·3	3·6	2·1	2·0	·46	·45
13	4·0	4·3	2·0	2·0	·46	·46
14	4·6	4·9	1·8	1·8	·45	·45
15	5·2	5·7	1·7	1·6	·46	·45
16	5·9	6·2	1·5	1·4	·46	·46
17	6·5	6·8	1·4	1·3	·45	·48
18	7·2	7·4	1·1	1·0	·47	·46
19	7·7	8·2	0·9	0·7	·46	·46
20	8·6	9·0	0·6	0·6	·46	·46
21	9·4	9·5	0·5	0·5	·47	·47
22	9·7	9·8	0·4	0·3	·46	·45
23	9·9	9·9	0·3	0·4	·45	·44
24	9·9	9·8	0·3	0·3	·44	·45
25	9·8	9·4	0·4	0·5	·45	·46
26	9·3	8·9	0·5	0·6	·46	·49
27	8·5	7·9	0·7	0·8	·47	·49
28	7·2	6·7	0·8	0·8	·48	·48
29	6·2	5·6	0·8	0·9	·48	·49
30	5·2	4·7	1·0	0·9	·49	·48
31	4·3	3·8	1·2	1·2	·49	·48

JUNE.

	Longitude.		Latitude.		Parallax.	
	Noon.	Mid.	Noon.	Mid.	Noon.	Mid.
	″	″	″	″		
1	+3·2	+2·9	+1·3	+1·4	+·47	+·50
2	2·6	2·2	1·4	1·5	·48	·48
3	2·0	2·0	1·6	1·6	·49	·47
4	1·9	2·0	1·6	1·6	·47	·47
5	2·0	1·9	1·6	1·7	·48	·45
6	1·8	1·9	1·7	1·7	·45	·47
7	1·8	2·1	1·7	1·7	·46	·46
8	2·3	2·6	1·7	1·8	·47	·46
9	2·7	3·1	1·9	1·9	·46	·46
10	3·5	3·8	1·8	1·8	·45	·45
11	4·1	4·3	1·7	1·7	·45	·45
12	4·8	5·0	1·6	1·6	·46	·46
13	5·4	5·7	1·5	1·5	·45	·46
14	6·0	6·3	1·4	1·4	·46	·46
15	6·8	7·0	1·3	1·1	·45	·46
16	7·2	7·6	1·1	1·0	·46	·46
17	7·8	8·2	0·8	0·8	·48	·49
18	8·5	8·6	0·8	0·6	·49	·48
19	8·9	9·2	0·6	0·6	·50	·49
20	9·3	9·5	0·6	0·6	·48	·49
21	9·5	9·6	0·6	0·7	·48	·47
22	9·6	9·6	0·7	0·7	·48	·47
23	9·5	9·5	0·7	0·7	·47	·46
24	9·4	9·2	0·7	0·8	·46	·47
25	8·7	8·4	0·7	0·8	·48	·46
26	7·9	7·4	0·7	0·7	·46	·46
27	6·9	6·5	0·7	0·7	·45	·45
28	6·0	5·5	0·7	0·7	·47	·47
29	5·1	4·7	0·7	0·7	·46	·45
30	4·2	3·8	0·8	0·9	·45	·45

JULY.

	Longitude.		Latitude.		Parallax.	
	Noon.	Mid.	Noon.	Mid.	Noon.	Mid.
	″	″	″	″		
1	+3·4	+3·1	+1·0	+1·0	+·46	+·44
2	2·9	2·7	1·2	1·2	·46	·45
3	2·6	2·6	1·3	1·3	·44	·44
4	2·6	2·6	1·3	1·4	·45	·44
5	2·7	2·9	1·4	1·4	·46	·45
6	3·0	3·0	1·5	1·6	·45	·45
7	3·4	3·7	1·6	1·6	·45	·45
8	3·9	4·2	1·7	1·6	·44	·45
9	4·5	4·8	1·6	1·5	·45	·46
10	5·1	5·3	1·5	1·4	·46	·46
11	5·7	5·9	1·3	1·2	·45	·47
12	6·2	6·5	1·2	1·1	·48	·48
13	6·7	6·9	1·2	1·0	·48	·46
14	7·3	7·6	1·0	1·1	·45	·48
15	7·8	8·1	1·0	1·0	·48	·48
16	8·2	8·4	1·0	1·1	·47	·47
17	8·5	8·7	0·9	1·0	·48	·48
18	8·8	8·8	1·0	1·0	·47	·49
19	8·8	8·9	1·1	1·1	·48	·48
20	8·9	8·8	1·1	1·1	·48	·48
21	8·8	8·8	1·0	1·0	·48	·48
22	8·7	8·7	0·9	0·9	·46	·47
23	8·5	8·4	0·9	0·9	·47	·47
24	8·2	8·0	0·8	0·8	·48	·47
25	7·7	7·3	0·7	0·6	·46	·47
26	6·9	6·5	0·7	0·7	·46	·45
27	6·0	5·5	0·7	0·6	·45	·46
28	5·1	4·7	0·6	0·5	·47	·47
29	4·3	4·1	0·6	0·7	·46	·47
30	3·7	3·5	0·6	0·7	·46	·46
31	3·2	3·1	0·8	0·9	·45	·44

AUGUST.

	Longitude.		Latitude.		Parallax.	
	Noon.	Mid.	Noon.	Mid.	Noon	Mid.
	″	″	″	″		
1	+3·1	+3·1	+0·9	+1·0	+·44	+·43
2	3·0	3·1	1·1	1·1	·43	·43
3	3·3	3·4	1·2	1·3	·43	·43
4	3·7	3·7	1·3	1·4	·44	·44
5	3·9	4·2	1·4	1·3	·45	·45
6	4·5	4·8	1·3	1·2	·47	·47
7	5·1	5·4	1·1	1·1	·46	·47
8	5·7	5·9	0·9	0·9	·46	·48
9	6·4	6·4	0·7	0·8	·49	·49
10	6·6	7·1	0·7	0·7	·49	·48
11	7·0	7·2	0·7	0·7	·49	·50
12	7·4	7·6	0·7	0·8	·51	·50
13	7·8	7·9	0·8	0·9	·50	·51
14	7·9	7·9	0·9	1·0	·50	·49
15	7·8	7·8	0·9	1·0	·50	·48
16	7·4	7·3	1·2	1·2	·48	·48
17	7·3	7·1	1·3	1·4	·46	·45
18	6·9	6·7	1·3	1·4	·45	·45
19	6·6	6·4	1·4	1·3	·44	·44
20	6·2	6·1	1·3	1·3	·46	·46
21	6·1	6·0	1·2	1·1	·47	·44
22	6·0	5·9	1·2	1·2	·46	·46
23	5·8	5·5	1·2	1·1	·45	·46
24	5·4	5·0	1·0	1·1	·45	44
25	4·7	4·5	1·0	1·0	·46	·47
26	4·2	3·9	1·1	1·0	·47	·46
27	3·6	3·5	1·0	1·0	·46	·46
28	3·3	3·0	1·0	1·0	·46	·46
29	2·8	2·7	1·1	1·0	·45	·44
30	2·5	2·6	1·1	1·0	·43	·43
31	2·6	2·7	1·0	1·0	·44	·43

SEPTEMBER.

	Longitude.		Latitude.		Parallax.	
	Noon.	Mid.	Noon.	Mid.	Noon.	Mid.
	″	″	″	″		
1	+2·8	+2·9	+0·9	+1·0	+·43	+·44
2	3·1	3·4	0·8	0·8	·45	·45
3	3·6	3·8	0·8	0·8	·44	·45
4	4·0	4·3	0·7	0·5	·44	·45
5	4·7	5·1	0·5	0·4	·46	·48
6	5·3	5·7	0·3	0·3	·46	·47
7	6·0	6·2	0·2	0·3	·48	·48
8	6·3	6·5	0·2	0·3	·48	·48
9	6·6	6·7	0·4	0·4	·50	·50
10	6·9	7·1	0·4	0·4	·52	·51
11	7·2	7·3	0·4	0·6	·52	·52
12	7·3	7·0	0·6	0·7	·53	·53
13	6·9	6·6	0·8	0·8	·52	·51
14	6·3	6·0	0·9	0·9	·50	·50
15	5·6	5·3	1·0	1·0	·47	·47
16	5·0	4·8	1·1	1·2	·47	·45
17	4·5	4·0	1·3	1·2	·44	·44
18	3·9	3·7	1·5	1·5	·43	·45
19	3·6	3·5	1·5	1·6	·45	·45
20	3·6	3·5	1·6	1·6	·46	·45
21	3·6	3·6	1·6	1·6	·46	·45
22	3·5	3·5	1·5	1·5	·46	·45
23	3·3	3·2	1·5	1·4	·43	·45
24	3·1	3·0	1·4	1·5	·44	·44
25	3·0	3·0	1·3	1·5	·45	·45
26	2·9	2·7	1·4	1·4	·44	·45
27	2·8	2·6	1·3	1·1	·45	·45
28	2·6	2·6	1·2	1·2	·44	·45
29	2·5	2·5	1·0	0·8	·46	·46
30	2·5	2·7	0·8	0·7	·45	·45

OCTOBER.

	Longitude.		Latitude.		Parallax.	
	Noon.	Mid.	Noon.	Mid.	Noon.	Mid.
	″	″	″	″		
1	+2·9	+3·0	+0·6	+0·4	+·46	+·44
2	3·2	3·6	+0·3	+0·1	·45	·45
3	3·8	4·1	0·0	−0·2	·45	·45
4	4·5	4·8	−0·3	0·3	·45	·47
5	5·2	5·4	0·3	0·3	·48	·47
6	5·8	6·1	−0·3	0·1	·48	·48
7	6·3	6·4	0·0	−0·1	·47	·50
8	6·5	6·7	0·0	+0·1	·49	·49
9	6·8	6·9	+0·2	0·4	·50	·49
10	6·9	7·0	0·4	0·6	·50	·50
11	6·8	6·6	0·8	1·0	·50	·51
12	6·3	5·9	1·0	1·1	·51	·50
13	5·4	5·1	1·1	1·0	·50	·49
14	4·8	4·6	1·1	1·1	·50	·49
15	4·3	3·7	1·1	1·1	·49	·48
16	3·7	3·4	1·0	1·2	·47	·47
17	3·1	2·7	1·3	1·5	·45	·46
18	2·6	2·5	1·5	1·7	·46	·46
19	2·5	2·6	1·7	1·7	·45	·45
20	2·6	2·7	1·7	1·8	·45	·45
21	2·7	2·8	1·7	1·7	·45	·44
22	2·6	2·6	1·6	1·6	·43	·44
23	2·5	2·6	1·6	1·6	·44	·44
24	2·7	2·8	1·5	1·5	·45	·44
25	3·0	3·1	1·5	1·4	·44	·45
26	3·1	3·1	1·4	1·4	·44	·45
27	3·1	3·0	1·2	1·2	·45	·44
28	3·0	2·9	1·1	1·0	·45	·44
29	3·1	3·1	0·8	0·8	·46	·46
30	3·4	3·5	+0·5	+0·3	·45	·45
31	3·6	3·8	0·0	−0·1	·45	·45

NOVEMBER.

	Longitude.		Latitude.		Parallax.	
	Noon.	Mid.	Noon.	Mid.	Noon.	Mid.
	″	″	″	″		
1	+4·0	+4·2	−0·2	−0·3	+·46	+·47
2	4·4	4·6	0·4	0·4	·47	·47
3	5·0	5·4	0·4	0·4	·48	·49
4	5·7	6·0	0·2	0·2	·50	·47
5	6·2	6·4	0·2	−0·2	·49	·49
6	6·4	6·4	−0·1	+0·1	·48	·50
7	6·5	6·5	+0·2	0·4	·50	·51
8	6·6	6·5	0·6	0·8	·51	·50
9	6·4	6·1	1·0	1·1	·50	·47
10	5·8	5·2	1·2	1·4	·46	·46
11	4·8	4·5	1·4	1·4	·47	·47
12	4·3	4·0	1·4	1·3	·48	·47
13	3·8	3·7	1·3	1·2	·49	·48
14	3·5	3·2	1·3	1·3	·47	·47
15	2·9	2·7	1·3	1·5	·47	·45
16	2·5	2·4	1·6	1·7	·45	·45
17	2·3	2·4	1·8	1·9	·44	·47
18	2·6	2·7	1·8	1·8	·46	·46
19	2·8	2·8	1·7	1·8	·45	·45
20	2·8	2·7	1·8	1·6	·46	·44
21	2·8	2·7	1·7	1·6	·45	·44
22	3·0	3·2	1·5	1·4	·44	·45
23	3·3	3·5	1·4	1·3	·45	·44
24	3·6	3·7	1·3	1·2	·44	·44
25	3·7	3·7	1·0	0·9	·45	·44
26	3·6	3·7	0·8	0·6	·45	·45
27	3·7	3·9	0·4	+0·3	·46	·46
28	4·0	4·0	+0·2	0·0	·46	·46
29	4·3	4·3	−0·1	−0·1	·47	·46
30	4·3	4·3	0·2	0·2	·48	·48

DECEMBER.

	Longitude.		Latitude.		Parallax.	
	Noon.	Mid.	Noon.	Mid.	Noon.	Mid.
	″	″	″	″		
1	+4·3	+4·4	−0·2	−0·2	+·50	+·48
2	4·6	4·8	−0·2	−0·1	·48	·49
3	5·1	5·2	0·0	+0·1	·49	·49
4	5·5	5·4	+0·2	0·1	·48	·49
5	5·5	5·5	0·2	0·2	·48	·51
6	5·6	5·5	0·4	0·3	·51	·51
7	5·4	5·5	0·4	0·5	·50	·51
8	5·4	5·3	0·7	0·8	·50	·46
9	5·1	4·9	0·9	1·0	·49	·48
10	4·5	4·1	1·0	1·1	·46	·48
11	3·9	3·8	1·2	1·3	·45	·46
12	3·5	3·4	1·3	1·4	·47	·48
13	3·4	3·3	1·4	1·4	·46	·45
14	3·1	3·1	1·6	1·6	·44	·45
15	3·2	3·0	1·7	1·7	·45	·45
16	3·1	3·1	1·8	1·8	·44	·47
17	3·3	3·4	1·8	1·8	·46	·47
18	3·6	3·6	1·7	1·8	·47	·46
19	3·7	3·8	1·7	1·6	·45	·44
20	3·9	3·8	1·6	1·6	·44	·45
21	3·9	4·0	1·6	1·3	·44	·45
22	4·2	4·2	1·3	1·2	·43	·44
23	4·5	4·6	0·9	0·7	·43	·45
24	4·6	4·6	0·6	0·6	·43	·44
25	4·6	4·5	0·5	0·3	·44	·44
26	4·5	4·6	+0·2	+0·1	·45	·44
27	4·8	4·8	0·0	−0·1	·44	·47
28	4·9	4·9	0·0	−0·1	·47	·46
29	4·8	4·9	0·0	0·0	·46	·47
30	4·9	4·8	0·0	+0·1	·48	·48
31	4·6	4·7	+0·1	+0·2	·48	·49

The following Reports were received too late for insertion in the Annual Report of the Council.

Adelaide Observatory, South Australia.
(*Director, Mr. G. F. Dodwell, Government Astronomer.*)

Meridian and Time-Service.—The transit-circle telescope was used for the observation of clock stars of the Sydney zone (-52° to -64°). The number of observations is shown in the following table :—

	Observations in R.A.	Declination.
Intermediate stars	1500	1497
Clock stars for time and longitude .	249	
Azimuth stars	99	
Totals . . .	1848	1497
Determinations of collimation .	134	
,, ,, level . .	161	
,, ,, nadir point .	111	

Azimuths were also determined from high and low stars north of the zenith.

A study of the changes of level of the transit-circle telescope during the last twenty-four years shows a remarkable effect of unsymmetrical exposure to the weather and surface drainage on the E. and W. sides of the transit buildings. Owing to this cause the ground on the W. side expands more than the E. side in the winter, raising the level on that side, and contracts more than the E. side in the summer, the west end being relatively low. The rainfall and temperature data enable these changes, which lag slightly behind the meteorological causes, to be predicted with a fair degree of approximation. An experiment made this summer of watering the ground on the W. side two mornings each week has met with an immediate result in the direction expected, and it now appears possible to maintain the level of the transit-circle telescope in equilibrium, by supplying the balance of moisture required on the E. or W. side as the case may be.

The time-ball at the Semaphore was dropped at 1 p.m. daily (Sundays, Christmas Day, and Good Friday excepted), the fall of the ball being recorded at the Observatory by return signal automatically sent by the time-ball mechanism. The time-ball at Port Pirie has also been dropped at 1 p.m., this service being under the charge of the Assistant Harbour-master at the port, who receives a time-signal at noon daily from the Observatory, by means of which the time-ball clock is corrected. On April 11, a wireless receiving apparatus was installed at the Port Pirie Time-Ball Station, and the midday wireless time-signals from Adelaide have been used instead of land-line signals for correcting the clock. The wireless time-signals for shipping (those of the Onogo system)

have been sent at midday and midnight each day by the Adelaide Radio Station, which is in telegraphic communication with the Observatory. The signals proceed from the Observatory mean-time clock and are automatically transferred to the wireless circuit.

Field Work.—Latitude and longitude stations were established at four stations, Nairne, Goolwa, Port MacDonnell, and Mt. Ruskin. Star observations were taken by the almucantar method with an altazimuth instrument provided with first-class level on the vertical circle. Longitude signals from the Observatory *via* the Adelaide Radio Station were received by means of a portable wireless receiving outfit. At Mt. Ruskin, where, for some unknown reason, the Adelaide wireless signals could not be heard, though sent on usual strength, the Melbourne radio time-signals were made use of. A special set of time-signals sent from the Melbourne Observatory to the Hector Observatory, New Zealand, was also received with the wireless receiving installation at the Adelaide Observatory. The preparation of tables for facilitating almucantar computations for observations in latitudes 26° to 40° S. has been commenced. The Surveyor-General is assisting in this work, and Mr. C. J. Merfield, of Melbourne, has lent a set of manuscript tables which have been of great assistance.

Terrestrial Magnetism.—Observations of magnetic declination, horizontal intensity, and inclination have been made at Nairne, Goolwa, Port MacDonnell, Mt. Ruskin, Winceby Island, and Althorpe Island. Inclination was also determined at Port Pirie, supplementing the other observations previously made. Preliminary observations of magnetic disturbances reported to exist in Spencer's Gulf were made during a voyage of the Harbours Board steamer *Conqueror* in that gulf in April. The results showed that the values of E. declination are from one to two degrees above the normal values in the upper half of the gulf. A magnetic survey of an area of the bed of the River Torrens at Campbelltown was made in an attempt to locate a pumping engine and suction pipe carried away in a flood during the previous winter. Professor Kerr Grant supplemented these observations by the use of a Hughes induction balance. The combined result was the recovery of a number of pieces of the suction pipe and the location of a large "island" of magnetic disturbance over the site of the former pump hole, a large excavation now filled with flood deposit. Owing to the rapid inflow of water, digging for the engine in this site was postponed till a future dry season. Professor Grant also co-operated in the latitude, longitude, and magnetic observations at four of the field-stations previously mentioned.

Seismology.—The Milne Seismograph was continuously at work during the year, and tabulated records were sent as usual to the Shide Observatory, Isle of Wight.

Miscellaneous.—The Board of Visitors held their annual visitation on December 6. The equatorial telescope has been used principally in connection with the public visits to the Observatory on Friday evenings.

Sydney Observatory.

(*Director, Mr. W. Ernest Cooke, Government Astronomer.*)

The outstanding feature of the year has been the recommencement of the systematic meridian observations under improved conditions. These were started on March 11, the working programme being the officially recognised list of intermediate stars for the region 51° to 65° south declination. It is proposed to observe each of these at least ten times, using for standards those stars of the international list of fundamentals which have been recently well observed at the Cape, of which the proper motions are known. Up to the end of the year the two observers, Messrs. Raymond and Cranney, took 4728 complete observations of 892 stars, and the reductions are up to date.

This has naturally delayed progress to some extent on the reductions of the old transit observations, but we have had a certain amount of energy available which will soon be required for measuring fresh astrographic plates, and so the older routine work has been kept steadily going.

For the rest, the concluding words of our last report (*M.N., R.A.S.*, **78**, 401, 1918) are still applicable.

The astrograph is practically completed for a fresh start, but this is still being delayed by circumstances beyond the astronomer's control.

No definite programme has yet been arranged for the 11½-inch equatorial, for two reasons :—

(1) The dome is almost unworkable.

(2) The astrographic work, both past and future, has first claim ; and until the new observational programme is in full swing it is unwise to fritter away our limited energy upon anything else. The keynote of the observatory at the present time is concentration upon the astrographic work, including of course the meridian portion.

The usual routine duties, including University lectures, time-service, seismograph, reception of visitors, etc., have been maintained as heretofore.